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AN EMPIRICAL EXAMINATION OF THE ROBUSTNESS OF ARBITRAGE FACTORS

by

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M.S., Air Force Institute of Technology, 1992

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AN EMPIRICAL EXAMINATION OF THE ROBUSTNESS OF

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This Dissertation is Dedicated to My Family:

Karen, Ryan, and Allison

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Randall Barry Howard

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CHAPTER 1

INTRODUCTION

After thirty years of vigorous research, there is still little agreement in the field of asset pricing theory. Shanken and Smith (1996) sum up the vast amount of empirical research on asset pricing models by saying, "Although we have learned much about the cross-sectional and time-series properties of returns and have developed sophisticated statistical methods to increase the power of the tests, numerous unanswered questions remain." Two of the most fundamental, yet unanswered, questions are: How many factors are there? and What are those factors?

The two primary equilibrium, expected return models are the Capital Asset Pricing Model (CAPM), developed almost simultaneously by Sharpe (1964), Lintner (1965), and Mossin (1966), and the Arbitrage Pricing Theory (APT), introduced by Ross (1976, 1977). The CAPM is a one factor model that states that the equilibrium rate of return on any asset is a linear function of the asset's covariance with the market portfolio. The APT, on the other hand, is a multifactor model.

Although, while initially the CAPM received widespread support, it has subsequently been rejected in numerous papers. For example, Roll (1977) provides his seminal critique concerning the testability of the CAPM. The finance literature has documented numerous anomalies in return series that cannot be explained by the CAPM.[1] Perhaps the strongest evidence against the CAPM is Fama and French's

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[1]The three main anomalies are: the day of the week effect, the small firm effect, and the January effect. See for example Banz (1981), Basu (1983), Keim (1983), and Bhandari (1988). Megginson (1997) provides an excellent review of these anomalies

(1992) conclusion that beta has little significant power in explaining the cross-sectional variation in asset returns. Despite this evidence, the CAPM remains the preferred model of choice for many financial practitioners and by academicians for classroom use in undergraduate and MBA programs.

Jagannathan and Wang (1996) suggest, as do many others, that the CAPM survives for several reasons. First, most of the other asset pricing models do not fare much better. Second, the theory behind the CAPM has an intuitive appeal that the other models lack. More fundamentally, there is some debate as to whether or not the evidence against the CAPM is economically significant.[2]

Recently, numerous papers have argued that multifactor models may provide a better description of average returns. Among the multifactor models, one can argue that Ross's Arbitrage Pricing Theory (APT) stands out as the heir apparent to the CAPM.[3] The APT is attractive for several reasons. It is based on relatively weak assumptions and allows for multiple sources of systematic risk. The market portfolio plays no special role and the APT is valid on subsets of assets.

Perhaps the biggest obstacle associated with the APT is it's silence on the number of priced factors required and what they are. If the factors were actually known, then the APT would have the same intuitive appeal of the CAPM. Investors

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in his Chapter 3, page 120.

[2]Kothari, Shanken, and Sloan [KSS] (1995) suggest that many of the adverse findings depend on how one interprets the usual statistical tests. Amihund, Christensen, and Mendelson (1992) argue that return data is too noisy to reject the CAPM. Black (1993) argues that some of the rejections are a sample period effect. KSS (1995) suggests that Fama and French's (1992) conclusion on beta was due to bias in Compustat data.

[3]Although the APT is just one example of a multifactor model the terms are often used interchangeably in the literature.

are only willing to pay for risk that cannot be diversified away and there are several different priced factors.

The empirical and theoretical literature on the APT is extensive. The general consensus is that the APT outperforms the CAPM. Specifically, it explains most, but not all, of the anomalies associated with the CAPM, it explains a larger percentage of the cross-sectional variation in returns, and it has lower pricing errors associated with it. However, there is little agreement as to the number and nature of significant factors. For example, numerous researchers have shown that the number of significant factors is dependent on sample size, sorting schemes, time period, etc. One possible explanation for these observations is that the APT performs better because it is taking advantage of sample specific information. Although several papers have suggested this possibility (either directly or indirectly), there has been little empirical work in the area.

This dissertation seeks to answer a critical question regarding the APT. Are the returns actually generated by a k factor model, as the APT hypothesizes, or are we simply improving the explanatory power for a unique sample by adding extra independent variables? It could be that the APT fares better than the CAPM because the additional factors of the APT exploit information that is unique to the sample at hand, and are not really representative of a true, economy wide linear factor structure.

Previous tests of the APT concentrate on the significance of factors "in-sample". In contrast, I focus on whether or not the factors are pervasive. To illustrate the difference consider a portfolio consisting only of securities related to the mining of precious metals. Clearly one would not be surprised to find some measure of inflation is a "priced" or significant factor for this sample. This does not mean that all assets will be affected by inflation. I consider a factor to be pervasive if it affects subsets of the entire population.

If security returns are generated by a k factor model, the factors extracted from different groups of securities (for a given time period), should be asymptotically equal. If the factors are significantly different, then it must be that either the extraction technique is not valid or different groups of securities are affected by different factors. The latter of which violates a key assumption of the APT. Although the APT is valid on subsets (i.e., it does not require the entire universe of securities to be used), one would expect the factors to be the same if the samples are sufficiently large and drawn from the same population. Given that the factors are equal, then if the APT is correct, the market price of risk for each of the factors should also be the same.

Work in this area to date includes Brown and Weinstein (1983), Cho (1984), and Conway and Reinganum (1988). Brown and Weinstein attempt to compare factors extracted from distinct portfolios. Cho extracts only those factors that are common among portfolios. Conway and Reinganum use the technique of cross validation to determine if the factors contribute in explaining out-of-sample returns. Unfortunately, as will be discussed in the following chapters, these papers rely on small portfolios and questionable methodologies. Using extensive simulation data, I hope to demonstrate a fundamentally sound technique for directly comparing factors extracted from large portfolios of security returns.

Using factors extracted via Connor and Koraczyk's (1986, 1988) asymptotic principal components technique, I hope to show that if returns are generated by a k factor process the first k factors will be the same when extracted from different portfolios. I then will analyze large portfolios of security returns to see if they are generated by k pervasive factors. As an additional test of the APT, I will look at the estimated "price" of the factors when using different portfolios. As will be discussed in the following sections, macroeconomic proxies will be used for the factors when testing if the price is the same across portfolios. This test might also be useful in

determining which macroeconomic variables are the best proxies for the true but unknown factors.

This dissertation is organized as follows: Chapter 2 provides a brief theoretical review of the APT and provides some basic notation. A review of the pertinent literature is found in Chapter 3. Next, in Chapter 4, I discuss the testable hypotheses and methodologies. The various sample selection criteria and data sources are discussed in Chapter 5. Chapter 6 presents the empirical results of the study. Finally, Chapter 7 provides a summary of the results and conclusions, and offers some recommendations for future research.

CHAPTER 2

ARBITRAGE PRICING THEORY

The introduction of the Arbitrage Pricing Theory (APT) by Ross (1976, 1977) is considered by many academicians as one of the most significant developments in Finance. Its introduction has generated a large body of both theoretical and empirical literature. This chapter starts with a review of Ross's original development of the APT. Next, I discuss the major approaches used to implement tests of the APT. Then, I discuss the major theoretical extensions of the APT. Finally, I examine the testability of the APT. The following chapter reviews the pertinent empirical literature, focusing on those papers that examine the number or stability of the priced factors.

2.1 Initial Development

This section provides a brief review of Ross's (1976, 1977) original development of the APT. The APT assumes the normal perfectly competitive and frictionless market assumptions (e.g., no taxes or transaction costs, and infinitely divisible assets). It assumes that individual investors have homogeneous expectations about asset returns and that the random returns for all assets are generated by a linear k-factor model of the following form:[4]

[4]As in Ross's original development - no time subscript is used. In fact, the APT is valid even in an intertemporal setting.

$$\begin{aligned} \tilde{r}_i &= E_i + b_{i1}\tilde{f}_1 + \cdots + b_{ik}\tilde{f}_k + \cdots + \tilde{\epsilon}_i \\ &= E_i + b_i\tilde{f} + \tilde{\epsilon}_i \\ i &= 1, \cdots n \end{aligned} \tag{2.1}$$

where

$\tilde{r}_i$ = the rate of return on asset i

E_i = the expected return on asset i

b_{ik} = the sensitivity of asset i to the k^{th} common factor

$\tilde{f}_k$ = the mean zero, k^{th} factor common to the return on all assets.

$\tilde{\epsilon}_i$ = the mean zero, idiosyncratic return on asset i

Or, in a more convenient matrix notation:

$$\tilde{r} = E + B\tilde{f} + \tilde{\epsilon} \tag{2.2}$$

where $\tilde{r}$, E *and* $\tilde{\epsilon}$ *are* $n \times 1$, B *is* $n \times k$, *and* $\tilde{f}$ *is* $k \times 1$.

The main additional assumptions involve those necessary for a diversification argument, which requires that one is able to completely diversify away the idiosyncratic risk. This requires that n be much larger than k and that the noise vector, $\tilde{\epsilon}$, be sufficiently independent so that idiosyncratic risk can be eliminated in large portfolios. A sufficient, but not necessary, condition for this requirement is that the $\tilde{\epsilon}_i$'s be mutually stochastically uncorrelated, and have finite variances. This is the assumption originally used by Ross, although as we will see later, he notes that it can be weakened. This assumption, that the covariance matrix of the idiosyncratic return component is diagonal, gives us an exact factor structure.

The APT, as the name implies, is based on a simple no arbitrage argument. Any portfolio which uses no wealth and involves no risk must, on average, yield zero return. The development proceeds along the following lines. Let η represent an

n x 1 vector representing the wealth invested in each asset and let 1_n be an n x 1 constant vector of ones. Design a portfolio that requires zero investment (i.e., the total long position equals the total short position), giving us:

$$\eta' 1_n = 0 \tag{2.3}$$

We will also assume that the portfolio is well diversified

(*i.e.*, $\eta_i \approx \frac{1}{n} \; \forall_i = 1, \cdots, n$). With our earlier assumptions then, the law of large numbers will allow us to diversify away the idiosyncratic risk.[5] The return on this portfolio, R_p, is given by multiplying equation (2.2) by $\eta/$:

$$\begin{aligned} R_p &= \eta' \tilde{r} = \eta' E + (\eta' B)\tilde{f} + \eta' \tilde{\varepsilon} \\ &= \eta' E + (\eta' B)\tilde{f} \end{aligned} \tag{2.4}$$

Further assume that we engineer the weights of the portfolio so that it has zero systematic risk, giving us

$$\eta' b_i = 0 \quad \forall \; i = 1, \cdots, k \tag{2.5}$$

or $\quad \eta' B = 0$

and then from equation (2.4) we have

$$R_p = \eta' E \tag{2.6}$$

Now, the random return is a certainty and to rule out arbitrage, must be equal to zero, giving us

$$\eta' E = 0 \tag{2.7}$$

Therefore, if η is orthogonal to 1_n [equation (2.3)] and η is orthogonal to B [equation (2.5)], η must also be orthogonal to E [equation (2.7)]. From linear algebra, this

---

[5]In the strict sense we have $\lim_{n \to \infty} \eta' \tilde{\varepsilon} = 0$, for now we will assume $\eta' \tilde{\varepsilon} = 0$, and later we will adjust our results accordingly.

will only occur if E can be written as a linear combination of l_n and the b_i's. This gives us the ex-ante expected return form of the APT

$$\begin{aligned} E_i &\approx \lambda_0 + \lambda_1 b_{i1} + \cdots + \lambda_k b_{ik} \\ &\approx \lambda_0 + \lambda b_i \end{aligned} \tag{2.8}$$

Or in matrix notation

$$E \approx \lambda_0 1n + \lambda B \tag{2.9}$$

Equations (2.8) and (2.9) are not strict equalities, since $\eta'\tilde{\epsilon} = 0$ only in the limit as n approaches ∞. E_0 can be interpreted as the return on an asset with zero systematic risk (i.e., a risk-free rate or the return on a zero-beta asset). As for the λ's, consider an asset with unit risk to factor j (i.e., $b_{ij} = 1$) and zero risk to all other factors (i.e., $b_{ik} = 0 \; \forall \; k \neq j$). From equation (2.9), the expected return on this asset will be

$$E_i \approx \lambda_0 + \lambda_j \tag{2.10}$$

giving us

$$\lambda_j \approx E_i - \lambda_0 \tag{2.11}$$

so the λ's are, in essence, risk premium's. Admati and Pfleiderer (1985) demonstrate that the λ's can be interpreted as excess returns on portfolios perfectly correlated with the factors if a riskless asset and such portfolio's exist.

In summing up this development of the APT, it is worth highlighting some of the strengths of the APT. First, the APT makes no assumptions about the underlying distribution of security returns. Nor does it make any assumptions about individual investors utility functions (other than nonsatiation). In contrast, the CAPM requires either normally distributed returns or a quadratic utility function for investors; neither of which are very appealing assumptions. Finally, the market portfolio plays no special role in the APT and the APT is valid on subsets. These strengths suggest that

the APT is a good alternative to the CAPM.[6] Unfortunately, the APT is not without serious flaws of it's own. Certainly the most fundamental problem is the APT's silence on the number and nature of the factors. This weakness is important for two reasons. First, as Megginson (1997) points out, it makes it almost impossible to operationalize the model in a corporate finance setting. Secondly, it causes a multitude of econometric problems which will be discussed in detail in the following chapter. Basically, before any tests can be performed in an APT setting, we have to identify the factors, their associated loadings, and their prices.

2.2 Factor Specification

The primary weakness of the APT is it's silence on the number of factors and what the factors are.[7] In addition to making the theory somewhat ambiguous, this causes numerous challenges when designing tests of the theory. The first challenge is that one must somehow identify the factors. Connor (1995) classifies multifactor models into one of three groups based on how they tackle this identification problem. He calls these groups: 1) macroeconomic factor models, 2) fundamental factor models, and 3) statistical factor models. These groupings are not entirely distinct and some researchers have looked at models that use two or more of the different types of factors. For example, Fama and French (1993) combines elements of fundamental and macroeconomic factor models and Burmeister and McElroy (1988) use statistical and macroeconomic factors. The following sections will highlight the relative

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[6]Another interesting strength of the APT is first discussed by Jarrow (1988). He argues that the APT is based only on ordinal utility theory, rather than cardinal utility theory as in the CAPM. This allows for systematic violations of the strong independence axiom as some argue occurs in the marketplace. He suggests that the most restrictive assumption of the APT is the linear factor hypothesis.

[7]In fact, Fama (1997) argues that determining the number of factors in the APT or the ICAPM is hopeless!

strengths and weaknesses of each of these types of models. Although this research will rely on statistical and macroeconomic factor models, a brief review will be given for each of the three types.

2.2.1 Macroeconomic Factors

One way to tackle the identification problem is to simply pre-specify the factors. Economic theory offers insight as to what factors might systematically affect the return on a security and the plethora of recorded variables offer observable proxies for these factors. The primary advantage of this technique is that it gives an intuitive appeal to the APT, as the factors now have some economic meaning. It also avoids some econometric problems associated with using estimated factors. Since we actually observe the proxies, we can conduct individual t-tests for whether or not the factors are priced. As we will see in the next chapter, many of the other techniques do not admit individual tests for significance.

Unfortunately, the use of pre-specified factors causes some unique econometric problems. First, each of the proxies has to measure the unanticipated movement (or innovation) of the relevant factors. If one simply used the underlying variable, without accounting for expected movement, it would result in an errors-in-variables problem, thus rendering the usual statistical tests useless. Also, many macroeconomic variables are correlated and this can cause other econometric challenges.[8] Finally, most macroeconomic data is only available monthly thus eliminating the use of weekly or daily returns.

This technique is also dangerous in that one must arbitrarily choose which variables to use. Fama (1991) points out that these approaches offer much flexibility and promise, but there is the danger that the relations are simply due to special features of a particular sample ("factor dredging"). For instance, it might be that

[8]Significant correlation among the factors can also diminish the intuitive appeal of this technique.

during an oil embargo a variable associated with oil prices is significant, yet in other periods it might not be.

Not only do we have to choose the macroeconomic proxies we believe will impact prices, we also have to align the variables with the stock return data. In other words, how do the proxies relate to the factors in a time series sense. Consider a macroeconomic variable whose value for the previous month (say April), is announced on the 15th of this month (May). Is the asset's return driven by the new information or the actual underlying effects of the macroeconomic factor? If it is more of an underlying relationship, then May's return will be driven by May's observation (even though it won't be released until June). Then one must decide what to do when the variable is updated or the technique for calculating it is revised.

Finally, these types of models do not seem to perform as well as other multifactor models do. Connor (1995) using an explanatory power approach, finds that the fundamental and statistical factor models consistently outperform the macroeconomic factor models.

2.2.2 Fundamental Factors

Fundamental factor models use observed company parameters (e.g., book to market value) as factor betas. These methods then set up mimicking portfolios to identify the extra return associated with a unit exposure to each factor.

Connor (1995) find that statistical and fundamental factor models significantly outperform macroeconomic factor models, and that fundamental factor models slightly outperform the statistical factor models. Connor suggests that the fundamental factor models outperform the statistical factor models because they include additional information. The statistical factor models maximize explanatory power based solely on returns data. Whereas the fundamental factor models incorporate firm specific variables such as book-to-market value, firm size, and dividend yield. Additional information is also often provided in the form of numerous industry dummies.

Since fundamental factor models are not used in this research, they will not be reviewed in the following chapter.[9] However, given the recent visibility of these types of models, it is worthwhile to briefly discuss them. Fama and French (1992, 1993) develop a three factor model consisting of size, book-to-market value, and a broad market index. They suggest that their model works because the fundamental, firm characteristics proxy for two unknown, underlying risk factors. As such, fundamental factors may play an important role in searching for answers about the APT. If we can find firm characteristics that seem to proxy for factors, these characteristics might offer clues as to the nature of the underlying factors. Specifically, they might lead us to those macroeconomic variables that proxy for the factors.

Unfortunately, the evidence on Fama and French's model is mixed. Kim (1995) suggests that Fama and French's results are due to an errors in variable problem. In a recent paper, Daniel and Titman (1997) argue that there is no risk factor associated with high or low book-to-market firms. They also argue that there is no return premium associated with any of Fama and French's (1993) factors. They assert that the strong covariances among firms with high book-to-market value are due to similar firm characteristics (e.g., same industries) and not common exposure to a priced risk factor. Knez and Ready (1997) argue that Fama and French's result on size is due to a few extreme outliers.

2.2.3 Statistical Factors

Statistical factor models use various statistical technique to extract the factors from cross-sectional and time series samples of security returns. The primary strength of these techniques is that one does not have to make arbitrary choices about what variables to include. The data itself determines the factors. There are

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[9]Examples of empirical papers that use a fundamental factor model include Rosenberg (1974), and Beckers, Grinold, Rudd, and Stefek (1992).

numerous techniques for "extracting" the factors from the data, and each of these techniques has its associated strengths and weaknesses. This review will focus on two major techniques. The first approach, used in many early papers, involves some form of maximum-likelihood factor analysis to extract the factors and their loadings from the sample covariance matrix of asset returns. The second approach involves an asymptotic principal components technique, due to Connor and Korajczyk (1986, 1988) that is similar to factor analysis but allows one to use large numbers of securities. These techniques will be reviewed in detail in the following chapter.

2.3 Theoretical Extensions of the APT

As with the CAPM, it was not long before academicians began questioning the validity of the APT. Most of these questions revolved around the approximate nature of the pricing relation. Shanken (1982) points out that equation (2.9) is an approximate pricing relationship, not an equilibrium model that must be satisfied by all assets.[10] Consequently, it should price most assets with negligible error, however, it allows for arbitrarily large deviations for a small number of assets in any finite economy. Not surprisingly, many researchers began to look at the magnitude of the pricing error and offered alternative derivations based on differing assumptions. A comprehensive bibliography of these early theoretical contributions can be found in Dybvig and Ross (1985). Some of the key results are discussed in the following paragraphs.

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[10]Actually, the static APT and CAPM are not true "pricing" models. By pricing models in this context we are talking about the ability to determine expected returns. An example of a true "pricing" model, one that actually can price securities, is the intertemporal version of the APT as discussed in Roll and Ross (1980), or the more general model of Cox, Ingersoll, and Ross (1985).

2.3.1 Approximate Factor Structure

Chamberland and Rothschild propose an approximate factor structure that is a much weaker assumption than Ross's strict factor structure. As mentioned earlier, a strict factor structure implies that the idiosyncratic components of asset returns have zero correlation across assets. Connor and Korajczyk (1993) offer an intuitive example of what a strict factor structure implies. One might easily agree that awarding a large defense contract to one aerospace firm might affect several aerospace firms. A strict factor structure would require this industry-specific risk to be a universal factor. Chamberland and Rothschild point out that it is unlikely any large group of securities will have a usefully small number of factors given a strict factor structure. In an approximate factor structure, the idiosyncratic risks can be correlated and hence the idiosyncratic covariance matrix need not be diagonal. So, industry-specific uncertainty, for example, does not have to be a pervasive factor. Instead, it will just show up as an off-diagonal term in the covariance matrix of the idiosyncratic risks.

Chamberland and Rothschild (1983) show that if the covariance matrix of the asset returns has k unbounded eigenvalues, then an approximate factor structure exists and it is unique. They also show that as the number of securities grow, the k eigenvectors associated with the k unbounded eigenvalues will asymptotically converge and play the role of the factor loadings. Connor (1984) and Ingersoll (1984) also weaken Ross's original assumptions and derive results similar to those of Chamberland and Rothschild. Grinblatt and Titman (1985) verify the Chamberlain and Rothschild results and show that any economy that satisfies an approximate factor structure can be transformed, without altering the portfolios investors hold, to an equivalent exact factor structure.

2.3.2 *Pricing Bounds*

Ross (1976), in his original development of the APT, showed that the sum of squared approximation errors in equation (2.9) is finite as the number of securities approach infinity. Subsequently, Huberman (1982) looks at the no arbitrage condition and provides conditions under which an economy will allow idiosyncratic risk to be completely diversified away. Chamberlain (1983) provides a pricing bound for the approximate factor structure model. Connor (1984) shows that the sum, across assets, of squared deviations goes to zero as assets are added. Shanken (1992) shows that a finite pricing bound holds when the factors are replaced by proxies that are sufficiently close to the true factors. Each of these papers makes additional assumptions, but none of the assumptions are overly restrictive. For example, many assume that there are portfolios that mimic the factors, that some agent holds a well diversified portfolio, that there are many assets, and that all assets are in positive supply.

Whereas the preceding papers all developed bounds for pricing errors among all assets, Craig and Malkiel (1982) give an intuitive explanation as to why the pricing error for individual assets should be small. Connor (1984), Dybvig (1983), and Grinblatt and Titman (1983) actually derive specific expressions for the pricing error for individual assets. Connor, and Grinblatt and Titman assume investors are risk averse and returns are distributed multivariate normal. Dybvig, assumes the market portfolio is efficient. Again, these derivations involve additional assumptions, but the assumptions are still somewhat appealing, and the results show that relative to the measurement error in expected returns the pricing error is very small.

So far, every theoretical version of the APT has assumed homogeneous beliefs regarding the expectations generating process. Connor and Korajczyk (1986) update their equilibrium version of the APT to allow an infinitesimal number of informed investors. Handa (1991) shows that with information uncertainty the APT still holds

as an approximate pricing relationship. Further, he shows that the pricing error will be a decreasing function of the information available and an increasing function of firm size and idiosyncratic risk. This extends earlier work by Dybvig (1983) and Grinblatt and Titman (1983) who, assuming homogeneous beliefs, find that the upper bound is an increasing function of size.

2.3.3 Equilibrium Versions of the APT

Carrying the theory even one step further, several papers have developed an equilibrium version of the APT, where equation (2.9) becomes an equality. Chen and Ingersoll (1983) suggest the APT will price all assets correctly if there exists a portfolio whose return has no idiosyncratic risk and some expected utility maximizer (with a continuously differentiable, increasing, and strictly concave utility function) finds that portfolio to be optimal. Connor (1984), Chamberlain (1983), and Grinblatt and Titman (1983) develop equilibrium models based on the existence of a well-diversified portfolio on the mean-variance frontier and by placing restrictions, such as risk aversion, on the investor's utility functions.

Lee, Park, and Wei (1993) develop an equilibrium APT in the same spirit as Shanken's (1985) theory and the unified APT/CAPM of Wei (1988). Essentially these models show that even without assuming perfect diversity in the market portfolio, as done by the above papers, the market portfolio will enter the pricing relation as a priced risk. Chamberlain (1988) develops an intertemporal equilibrium model that links the APT with Merton's (1973) Intertemporal Capital Asset Pricing Model (ICAPM). While these equilibrium versions of the APT give an exact equilibrium, expected return relationship, they also open the door to a joint hypothesis problem similar in spirit to tests of the CAPM.

Finally, Handa and Linn (1991) develop an equilibrium APT in an economy where investors have heterogeneous beliefs and differing information sets. In a following paper, Handa and Linn (1993) suggest that betas estimated with past returns

are complete information betas and ignore the fact that bayesian investors will account for estimation risk (i.e., they do not ignore variation of information across assets). They argue that when using complete information betas, average return deviation will be correlated with any information proxy (e.g., size), and as the number of securities are increased, so will the number of factors. This finding has obvious implications on the anomalies literature.

2.4 Testability of the APT

Since the focus of this dissertation is on the viability of the APT, it is important to address the theory relating to the APT's testability. Shanken (1982) presents the first primary challenge to the testability of the APT. Shanken has two major contentions. First, as previously discussed, the APT is developed in an infinite economy, yet empirical tests, by definition, take place with a finite number of assets. Second, Shanken suggests that one can fundamentally alter the nature of the underlying factor structure by simply rearranging the underlying securities into equivalent portfolios.

Dybvig and Ross (1985) argue that Shanken's contentions are groundless. They suggest that the APT is robust to linear transformations and that the factors should remain stable. Grinblatt and Titman (1985) show that Shanken's argument about repackaging securities to hide factors is invalid, because the variance of the repackaged security will approach infinity and be correctly identified as a factor. Shanken (1985) however, maintains that the APT is not testable and that most researchers test predictions based on the relatively restrictive equilibrium APT, yet use the assumptions of the less restrictive approximate APT. Shanken argues the equilibrium versions are not testable because they, like the CAPM, involve the market portfolio.

Grinblatt and Titman (1987) suggest that there is a big difference between the global mean-variance efficiency of the market portfolio in the CAPM and the APT saying that for "subsets" of the economy, the proxy portfolio is locally mean-variance efficient relative to the assumed factor structure. They suggest that with techniques allowing for large numbers of securities and factors to be analyzed it is possible to construct a portfolio whose idiosyncratic risk is virtually eliminated.

More recently, Shanken (1992) and Reisman (1992) suggest that the approximate APT relationship is a tautology for any finite set of assets. If the pricing bound is known, then there is nothing to test. This is because the finite bound on the pricing error will absorb any error due to misspecification of the factors.

2.5 Summary

In summary, the theoretical work on the APT seems to indicate that it should hold, either approximately or in equilibrium under varying assumptions. The assumptions are not overly restrictive, especially for traded assets, and since the APT is valid on subsets, one can reasonably expect the APT to hold. The following chapter looks at the empirical literature concerning the APT.

CHAPTER 3

REVIEW OF THE LITERATURE

Whereas the previous chapter provided a brief theoretical review of the APT and its early development, this chapter focuses on the empirical literature surrounding the APT. Where necessary, theoretical papers will be discussed as they apply to new methodologies. The purpose of this chapter is not to provide a comprehensive review of every paper that has tested the APT. Rather, we want to highlight those papers that have made a significant development or will be directly applicable to the research at hand. Specifically, we concentrate on those papers that look at determining the number of priced factors and the stability of these factors and prices. Connor and Korajczyk (1995) provide a thorough review of the APT and the empirical literature. As mentioned in the previous chapter, we focus on those papers that have used Connor and Korajczyks's asymptotic principal component technique or pre-specified macroeconomic variables. Other techniques are discussed as necessary. Chapter four will review specific methodology and testing as necessary.

3.1 Testing the APT

Before looking at the specific implementations and results of various tests of the APT, it is worthwhile to look at the task at hand. A theory can generally be tested by examining its assumptions or its implications. This research looks at both types of tests. First, regarding the assumptions, the APT's primary assumption is that returns are generated by a linear k factor function (equation (2.2) in the previous chapter),

$$\tilde{r}_t = E + B\tilde{f}_t + \tilde{\epsilon}_t \tag{3.1}$$

The other assumptions will be different, depending on which version of the APT one is investigating (e.g., strict versus approximate factor structure). Clearly, if one can demonstrate that returns are not generated by k pervasive factors, the other assumptions do not matter, as the APT will not hold.

The primary implication of the APT is it's pricing relationship (equation (2.11) in the previous chapter),

$$E = \lambda_0 + \lambda B \tag{3.2}$$

Tests of this implication have generally focused on: 1) how many factors are priced, 2) is the intercept term equal across groups, and 3) are any other variables priced? In addition, numerous papers have looked at how well the APT performs relative to the CAPM in either explaining the cross-section of returns or explaining the various pricing anomalies.

The t subscript has been added to equation (3.1) to illustrate that for most empirical analysis we will work with a time-series of t return observations (i.e., t = 1, ..., T).[11] We also will use an equilibrium version so that a strict equality holds. From these two equations, we notice that one must estimate all of the parameters using only the observed returns, since none of the other parameters are known. Most of the empirical procedures are based on a two-step procedure, similar to that which Fama and MacBeth (1973) used in their seminal analysis of the CAPM. In the first step we use time-series data to estimate the factors (f) and their beta's (**B**). If the factors are known *a-priori*, or proxies have been specified for them, as in the case of macroeconomic variables, the task is somewhat straightforward. Without knowing the factors we must extract the two terms (f and **B**) simultaneously, which is

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[11]As will be discussed later, some empirical tests also allow for time varying risk premia and beta's.

complicated by the fact that they enter into equation (3.1) multiplicatively. In the second step, we use cross-sectional regressions to estimate the factor's associated prices (λ). Factor analysis was the first technique used to extract the factors in the first of these two steps.

3.2 Factor Analysis

Although this effort will be relying primarily on Connor and Koraczyk's (1986, 1988) asymptotic principal component technique, it is useful to first review the technique of factor analysis. This technique is used in many of the early empirical tests of the APT and the various other techniques were designed to overcome some of the problems inherent in the factor analysis approach.

3.2.1 Theoretical Foundations

Roll and Ross (1980) is the seminal empirical work in this area. This section follows their theoretical development.[12] Roll and Ross start with Ross's original strict factor model, as given by equations (3.1) and (3.2), where $E[\tilde{\epsilon}_t \, \tilde{\epsilon}_t' \,] = \Sigma_D$, a diagonal matrix of own asset, idiosyncratic variances. They further assume that all of the relevant factors are accounted for, so that $E[\tilde{\epsilon}_t \mid \tilde{f}_t] = 0$. With this relationship, Roll and Ross show that the population covariance matrix, $\mathbf{V}$ = $E[(\tilde{r}_t - E)\,(\tilde{r}_t - E)']$ can be decomposed as follows:

$$\mathbf{V} = \mathbf{BLB}' + \Sigma_{\mathrm{D}} \tag{3.3}$$

where $\mathbf{B}$ is the matrix of factor loadings, $\mathbf{L}$ is the matrix of factor covariances, and Σ_{D} is as defined above. In essence, we have decomposed the overall covariance into systematic and non-systematic components. It is important to note that this specification is not unique. Consider any orthogonal matrix $\mathbf{G}$ (i.e., $\mathbf{GG}' = \mathbf{I}$).

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[12]The notation has been changed for consistency in this paper.

When one estimates **B** from the sample covariance matrix $\hat{V}$, all linear transformations **BG** will be equivalent. This can easily be seen from equation (3.3),

$$\begin{aligned} V &= (BG)(G'\Lambda G)(BG)' + \Sigma_D \\ &= BGG'\Lambda GG'B' + \Sigma_D \\ &= B\Lambda B' + \Sigma_D \end{aligned} \tag{3.4}$$

This problem makes comparison among factors, or the pricing of individual factors, almost impossible. For example, factors 1 and 2 could switch place or we could scale up factor j's loadings by a scalar g and scale down factor j by the same scalar g, and still obtain the same return relationship. This scaling, however would also have an influence on the price of factor j.

Roll and Ross suggest a maximum-likelihood approach to estimate **B**. Roll and Ross also assume the factors are orthogonal and scaled to have unit variance. Therefore **L** becomes the identity matrix. This condition can always be assured through scaling and rotating the factors.

After estimating the **B** matrix in this manner, the second step involves estimating the λ's. By substituting equation (3.2) into equation (3.1) we obtain

$$\begin{aligned} \tilde{R}_t = \tilde{r}_t - \lambda_0 &= B(\lambda + \tilde{f}_t) + \tilde{\epsilon}_t \\ &= B\lambda + B\tilde{f}_t + \tilde{\epsilon}_t \end{aligned} \tag{3.5}$$

Roll and Ross then note that the cross-sectional regressions will be biased by the time-series sample mean of the factors. As the sample size n increases, the bias should approach zero. Roll and Ross suggest the use of a generalized least squares (GLS) cross-sectional regression for each day t, where:

$$(\hat{\lambda}_t + \tilde{f}_t) = (\hat{B}'\hat{V}^{-1}\hat{B})^{-1}\,\hat{B}'\hat{V}^{-1}r_t \tag{3.6}$$

and $\hat{V}$ is the estimated covariance matrix of $B\tilde{f}_t + \tilde{\epsilon}_t$, which is the disturbance term in equation (3.5). Then the time-series average of the $\hat{\lambda}_t$'s will yield an unbiased estimate of λ since the expected value of $\tilde{f}_t$ is zero. The covariance matrix of $\hat{\lambda}$ will be

$$B'V^{-1}B \tag{3.7}$$

which, by design, will be diagonal. This means the estimated risk premia are independent.

One problem with the Roll and Ross approach is the computational burden associated with factor analysis of large matrices. For this reason Roll and Ross break their sample into groups of 30 securities and use factor analysis individually on each group. They find at least three, and possibly four, priced factors based on individual *t*-tests of significance. Roll and Ross also provide weak evidence that own-variance is not priced and that the intercept term is constant in the cross-sectional regressions. Both of these findings lend additional support to the APT.

3.2.2 Early Criticisms

Dhrymes, Friend, and Gultekin (DFG) (1984) offer three points of criticism concerning the Roll and Ross approach.[13] It is important to mention these criticisms since they have driven the development of many of the other techniques. They also cast a new light on interpreting results from these early papers. First, DFG point out that due to the linear transformation problem, individual *t*-tests are not valid. The only statistically sound test available is a joint test (an F-test) of whether or not all the factors together are significant. Next, they show that factor analysis on individual

[13]Roll and Ross (1984) take exception to many of DFG's criticisms. In a subsequent paper, however, Dhrymes, Friend, Gultekin, and Gultekin (1985) provide further evidence that factor analysis has serious flaws, and that the number of factors will increase with the number of securities.

groups is not equivalent to factor analysis on the entire sample. This is because of the lost off-diagonal covariance information. They also point out that each individual group may have different factors. They suggest that one needs to yield to the law of large numbers and use the entire sample.[14] Finally, they empirically demonstrate that with Roll and Ross's technique the number of significant factors will increase as the number of securities in the group is increased.

Overall, they think that there are more than five factors. They also point out that the most dominant factor's loadings remain somewhat stable as individual securities are added or deleted from a group; however the loadings for the other factors drastically change. Therefore it is difficult to make any inferences from the estimated loadings. This is also the first evidence that the nondominant factors (i.e., those factors other than the first), might be unique to the sample at hand and not indicative of an economy wide, pervasive factor.

3.2.3 Early Empirical Results

Numerous other researchers used the basic Roll and Ross methodology and at this stage, the evidence on the APT is mixed. This section will highlight some of these results and look at some extensions and modifications to the method of factor analysis.

Chen (1980) demonstrates that if you know the factor loadings for k portfolios, you can compute k factor loadings for any asset, say p, if you know the pth asset's covariance with the other k portfolios. This diminishes the computational burden of factor analysis; however, it forces a common factor model (which is estimated with a small number of securities) on all securities. In a subsequent paper, Chen (1983) compares the performance of the CAPM to the APT. He finds that the APT does a better job of explaining the cross-section of expected returns. Chen also

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[14]Unfortunately, computational limitations prohibit Roll and Ross from doing so.

shows that own variance and firm size have no explanatory power. Reinganum (1981), however, uses the same methodology, and finds that the APT is not able to remove the size anomaly.

Brown and Weinstein (1983), in a paper that might be considered the most closely related to this effort, propose a new approach to testing the APT.[15] Brown and Weinstein propose a test similar in spirit to that of Gibbons (1982) multivariate test of the CAPM. They call their test the bilinear hypothesis and argue that it is applicable to all asset pricing models. Basically, they make a crude attempt to compare factors among different groups. After adjusting their significance levels downward, due to a large sample size, they find evidence in support of a three factor APT, and reject a five and seven factor version.

Brown and Weinstein (1983) compare the factor structures across groups with an *F*-test that is based on the unconstrained and constrained error variances. Specifically, the statistic is based on the difference between the unconstrained residual sum of squares from factor analysis of two groups of 30 securities and the constrained residual sum of squares obtained from extracting the factors from the entire group of 60 securities. They find that more than three factors are necessary to explain a reasonable amount of the observed variation, but that the three factors that best explain the variation are the same across groups.

Brown and Weinstein claim this test directly compares the factors. In actuality it is a joint test that compares the predictive ability of the entire set of factors as a whole. If one of the factors explained a large portion of the variance their test might still say there were three (or more) common factors. In contrast, the test I propose allows for a more direct comparison of the factors, not their joint ability to explain the cross-sectional variation of returns.

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[15]This paper will be discussed in more detail in the following chapter.

Cho (1984) tackles the issue of factor comparability by employing inter-battery factor analysis. This technique extracts the factors from the inter-group sample covariance matrix instead of the full sample covariance matrix. This technique forces common factors among the groups but ignores a large amount of information in the overall covariance matrix. Cho groups the securities based on their SIC code and finds five or six common factors that appear to be responsible for generating daily security returns. Interestingly the number of factors does not appear to be related to the size of the groups. Specifically, Cho divides his data into 22 groups and then looks at the number of factors common to each pair-wise set of adjacent groups. The results indicate that the number of factors that are common between groups appears to be stable. Cho suggests there are approximately six factors. However, there is no guarantee that the factors are the same (e.g., the six factors common to groups one and two may be entirely different from the six factors he finds common to groups seven and eight).

Kryzanowski and To (1983) compare various methods of factor analysis and various methods for determining the number of significant factors. After extracting 50 factors from various portfolios they conclude that standard likelihood tests would suggest ten or more factors, but more stringent tests suggest there are less than five. They also make the important observation that while the first factor was associated with a large percentage of the securities, the remaining factors were not. Again, this appears to be evidence that the non dominant factors may be sample specific.

Other researchers have used factor analysis to examine returns from different markets. For example Beenstock and Chan (1986) look at the UK stock market. They find that a relatively high proportion of estimated expected returns is explained by the APT. They find weak evidence that more than 20 factors are significant. Beenstock and Chan find that the APT does subsume the size effect, but own variances are priced. Abeysekera and Mahajan (1987) also look at the UK market

and find mixed results. They find that the intercept terms are equal across groups, but the risk premia are jointly, insignificantly different from zero.

Gultekin and Gultekin (1987) show that empirical tests of the APT are very sensitive to the January effect. Basically they argue that the APT can explain the risk return relationship only in the month of January.[16] Cho and Taylor (1987), partially in an attempt to explain this finding by Gultekin and Gultekin, examine the month by month stability of the APT. Specifically they look at the stability of: 1) daily returns, 2) the covariance and correlation matrices, 3) the number of factors, and 4) the APT pricing relationship. Unfortunately, as they admit, many of their tests are very weak due to the non-uniqueness of factor loadings. Overall, they reject the APT and find that January does have a different pricing behavior.

Lehman and Modest (1988) develop a factor analysis technique for large samples that they claim is computationally more efficient than principle components. They find that after adjusting for the APT risk factors, dividend-yield and own variance play no role in estimating expected returns, whereas these variables have been shown to be related to excess returns in a CAPM model. They do not find, however, that the APT is able to account for the size effect.[17] Lehmann and Modest stress the importance of testing the expected return relationship with "out of sample" data (i.e., security returns that were not used to estimate the factor model). They argue that factor analysis has a tendency to over fit the data and therefore these "out

[16]Tinic and West (1984) document the same finding for the CAPM. The January effect is linked with many of the other anomalies, in particular, most or all of the size effect has been shown to occur in January.

[17]Most of the mispricing is concentrated in the smallest and largest firms, so they suggest that APT does fit most listed firms with little error.

of sample" tests are important.[18] This research will futher support this hypothesis by demonstrating that many of the apparently significant factors are unique to a particular sample.

In a similar vein, Conway and Reinganum (1988) use the cross-validation to test for a stable factor structure. Cross-validation involves fitting a model with one set of data and then seeing how well it predicts out of sample. If a factor is economy wide, or pervasive, it's inclusion will cause a decline in the out of sample prediction error. If the model is over-fit and the additional factor is sample specific the out of sample prediction error will remain the same or even increase. Conway and Reinganum are perhaps the first to clearly document that factors extracted from randomly selected securities may be specific to that particular sample. They find that common likelihood ratio statistics identify four or more factors whereas cross-validation suggests only one or two of these factors are pervasive. Conway and Reinganum base the adequacy of their technique on very limited simulation data. They look at the ability of their cross-validation technique to distinguish between pervasive and sample specific factors in one-factor and two-factor models. Their results may also be sensitive to the rotational indeterminacy of extracted factors.

Jobson (1988) proposes two indexes for goodness of fit that can be used in place of cross-validation. This allows the researcher to use more of the sample in estimating the model. The indexes are CA (criterion of Akaike), developed by Akaike (1974) and CS (criterion of Schwartz), developed by Schwartz (1978). Both the CA and CS index are penalty functions that penalize the likelihood statistic as more parameters are added. Jobson demonstrates that by using the CS index the number of factors found are similar to the results of Conway and Reinganum (1988).

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[18]Bower, Bower, and Logue (1984) showed that the APT performed better than the CAPM at explaining out of sample returns. However, their entire analysis was done using only utility stocks, and this will favor the APT.

Stambaugh (1988) argues that Conway and Reinganum (1988) based the adequacy of their technique on very limited simulation data. He suggests that more work needs to be done using simulated data to test the techniques based on various factor structures. Brown (1988) also questions the Conway and Reinganum (1988) study. Brown shows that if an economy has five equally important factors, factor analysis will identify one dominant factor (that will continue to grow with the number of securities) and four significant, but minor factors.[19]

3.2.4 Summary

As the above discussion illustrates, the early evidence on the APT is somewhat mixed. Clearly, it seems as if the APT does do better than the CAPM at explaining the cross-section of expected returns and it seems to account for some of the anomalies associated with the CAPM. Determining the number of priced factors seems to be the biggest obstacle. In fact, at this stage it appears to be more of an art than an exact science. With some exceptions, there appears to be a disturbing trend that as the sample size increases, so does the number of factors. It also appears that the first factor is somewhat dominant and the other factors may or may not be pervasive. Although, this result might be due to the rotational indeterminacy of factor analysis. It seems that simulation could easily be used to validate the ability of various techniques to accurately identify "priced" factors under varying factor structures. This would go a long way in helping the researcher understand the implications of the rotational indeterminacy.

As discussed in Chapter 2, the APT is valid on subsets, but since we are dealing with an equilibrium version of the APT, we need a large sample to diversify

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[19]Although Brown's (1988) work sheds much light on the empirical observation that one factor is usually dominant, it seems that if the other four minor factors are still pervasive they would be picked up in Conway and Reinganum's (1988) cross-validation technique.

away the idiosyncratic risk. Factor analysis limits the sample size and makes it hard to make inferences about economy wide, pervasive factors. This limitation is addressed in the next section.

3.3 Asymptotic Principal Components

The criticisms of DFG, and the mixed results of the early empirical studies, illustrate the motivation for researchers looking at alternatives to factor analysis in empirical tests of the APT. Connor and Koraczyk's (1986, 1988) asymptotic principal components technique is perhaps the most widely used of these alternatives. The technique of asymptotic principal components allows one to treat the factors as unobservables, like factor analysis does. However, it allows the researcher to overcome some of the computational problems associated with factor analysis. Much of the empirical work is based on a theoretical result from Chamberlain and Rothschild (1983) that was discussed in the preceding chapter.

3.3.1 Theoretical Foundations

Principal components analysis is simply another method of extracting the factors from returns data. The primary difference between principal components and factor analysis is that factor analysis estimates the idiosyncratic risks at the same time it estimates the factors; principal components on the other hand ignores the idiosyncratic risks. Principal components attempts to explain as much as possible of the entire cross-sectional variation in returns. Factor analysis only looks at the variation common to all securities. Principal component factors are based on the eigenvectors of the sample covariance matrix. Therefore, both factor analysis and traditional principal components require working with an *n x n* covariance (or correlation matrix). Connor and Korajczyk's (1986, 1988) gain in efficiency comes from their development of an asymptotic principal components technique for extracting the factors assuming an approximate factor structure.

With an approximate factor structure, the relationship in equation (3.3) becomes:

$$\mathbf{V} = \mathbf{BB'} + \Sigma_R \tag{3.8}$$

where B is the matrix of factor loadings, Σ_R is a matrix with uniformly bounded eigenvalues, and again we assume that the factors are orthogonal and scaled to have unit variance. Note however, that unlike with a strict factor structure, we allow Σ_R to have off-diagonal terms. It also assumes knowledge of the actual population covariance matrix of asset returns.

Connor and Korajczyk (1986) first show that statistics based on the observed sample covariance matrix will converge to the statistics based on the population covariance matrix. More importantly, they also show that one can use a $T \times T$ dimensional matrix, rather than the normal $n \times n$ dimensional sample covariance matrix. The following paragraphs will closely follow their original development.[20]

First, Connor and Korajczyk (1986) suggest the following empirical specification. They start with equation (3.5),

$$\begin{gathered} \tilde{R}_t = B(\lambda_t + \tilde{f}_t) + \tilde{\epsilon}_t \\ E[\tilde{\epsilon}_t | f_t] = 0, \quad E[\tilde{f}_t] = 0, \quad E[\tilde{\epsilon}_t \tilde{\epsilon}_t] = \Sigma_R \end{gathered} \tag{3.9}$$

The t subscript has been added to the λ's to note that this technique allows for time varying risk premia. Actually, with the Fama-Macbeth approach used by Roll and Ross (1980), one could argue that factor analysis also allowed for time-varying risk premiums. Next, assume we have observed returns on n securities for t periods.

Remembering that $\tilde{R}_t$, are excess returns (returns in excess of the riskless rate), the observed $n \times T$ matrix of excess returns is given by $R^N = r^n - 1_n r_f/$ where r^n is the $n \times T$ matrix of observed returns, and r_f is a T-vector of observed returns

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[20]Again, the notation is changed for consistency.

on the riskless asset. The risk-free rate can also vary with time. From equation (3.9) we have

$$R^n = B^n F + \varepsilon^n \tag{3.10}$$

where F is the k x T matrix of $(\lambda_t + f_t)$ and ε^n is the n x T matrix of realizations of ε_t and B^N is the n x T matrix of factor loadings.

Connor and Korajczyk will extract the principal components from equation (3.10). Factor analysis, on the other hand, uses equation (3.1) and extracts the factors from the sample covariance matrix $[(r^n - \hat{E})(r^n - \hat{E})']$ where $\hat{E}$ is estimated with the time series mean of r^n. This is a critical, but often overlooked, point. Any estimate of E is biased by the time series mean of the risk-premia, as can be seen by equation (3.2).

Ehrhardt (1987) is the first to discuss this issue in detail. He argues that most studies use a test statistic, based on the estimated errors, to determine whether there is a sufficient number of factors. In most of the factor analysis papers these estimated errors come from equation (3.1); Ehrhardt argues they should come from equation (3.5). Ehrhardt offers an intuitive explanation as to the differences of these approaches. With equation (3.1) the intercept term E_i is different for each asset, but the same in every period. With equation (3.5), however, the intercept is the risk-free rate which will be the same for each security but can be different for each period. So with equation (3.1) we can always drive the estimated residuals to appear as if they are distributed independently, with equation (3.5) however, we cannot.

Connor and Korajczyk (1986) next define

$$\Omega^n = (1/n)\, R^{n\prime} R^n \tag{3.11}$$

which, from equation (3.10) is equal to

$$\begin{aligned} \Omega^n &= (1/n)F'B^{n\prime}B^nF + (1/n)(F'B^n\epsilon^n + \epsilon^{n\prime}B^{n\prime}F) + (1/n)\epsilon^{n\prime}\epsilon^n \\ &= A^n + Y^n + Z^n \end{aligned} \tag{3.12}$$

Let G^N be the observed principal components matrix of Ω^n and H^n be the unobservable principal components of A^n. G^n is a k x T matrix of the k eigenvectors associated with the k largest eigenvalues of Ω^n. Connor and Korajczyk show that H^n will be a non-singular transformation of F (which is what they want to extract). Next, they show that Y^n approaches zero as n increases, and Z^n approaches σ^2 I_T for large n. So from equation (3.12)

$$\lim_{n \to \infty} \Omega^n = A^n + \sigma^2 I_T \tag{3.13}$$

and since the eigenvectors of a matrix do not change with the addition of a scalar matrix, the eigenvectors of Ω^n will approach those of A^n as n grows large. More formally, they prove,

$$G^n = L^n F + \Phi^n \text{ where } \underset{n \to \infty}{plim}\ \Phi^n = 0 \tag{3.14}$$

Therefore one can use $\mathbf{G}^n$ as an estimate of the factor loadings in the second stage cross-sectional regressions.

This technique effectively lets a researcher estimate the factors for a huge number of securities without having to break them into groups as Roll and Ross (1980) did. However, individual t-tests will still not be valid since the factors are only determined up to a non-singular transformation.[21] As with factor analysis, this technique assumes the value of k is known. Connor and Korajczyk suggest that one feasible test for the correct value of k is to make sure that the first k eigenvalues of Ω^n continue to grow as n increases, whereas the last T - k eigenvalues of Ω^n become equal (bounded).

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[21]There is some confusion on this matter in the literature. Shukla and Trzcinka (1980) state that since the approximate factor structure is unique, up to a scalar transformation, individual t-tests are valid.

3.3.2 Empirical Results and Extensions

In a subsequent paper, Connor and Korajczyk (1988) use their asymptotic principal components technique in an empirical study. They first offer a new procedure for estimating the matrix $\mathbf{G}^n$, which is an iterative technique similar to the use of weighted-least squares in regression. They point out that with large samples there is little gain from this technique but there might be gains with small samples. Presumably, one uses the asymptotic principal components technique to take advantage of using a large number of securities, so this iterative technique does not seem too important and will not be used in this research.

Empirically, they look at estimating factors and risk premiums for four non-overlapping five year periods, where each time period has between 1487 and 1745 securities (as compared to Roll and Ross's (1980) groupings of 30 securities). Connor and Korajczyk look at one factor, five factor, and ten factor models. They conclude that the five factor model is the most parsimonious and it performs better than a standard CAPM model in explaining anomalies such as the January-specific mispricing. However, even with the APT, the size effect remains.

Another very interesting result in this paper is Connor and Korajczyk's use of simulation to show that asymptotic principal components accurately estimates the factors. This approach will be one of the main techniques utilized in my analysis of factor stability. They simulate asset returns that correspond to an approximate five factor model and then extract the factors using their technique. They then regress each of the extracted factors on the full set of true factors. Comparisons cannot be made on a factor to factor basis because $\mathbf{G}^n$ converges to L^n F, not F. They find that their technique works well even if the idiosyncratic risks have a correlation coefficient of 0.9 (where a strict factor structure will have a coefficient of zero, and an approximate factor structure will be somewhere between zero and one).

Connor and Korajczyk (1988) also propose some new methods for testing restrictions implied by the approximate factor structure. For example, let "**a**" represent a vector of the intercept terms from the cross-sectional regressions of excess returns on the factors; the theory states that this vector should be exactly zero. They find that it is not identically zero but it seems to perform as well as or better than similar mispricing tests using a CAPM model.

Trzcinka (1986) looks at the behavior of the eigenvalues from a sample covariance matrix as the number of securities is increased. He finds that only the first eigenvalue grows without bound, but the first five appear to be growing more distinct. In a theoretical paper, Brown (1989) shows that the evidence shown in Trzcinka is consistent with an economy with k equally important factors.[22] Unlike, factor analysis however, the principal components solution is unique. Brown shows that the principal components solution will differ from the original factor structure by a particular rotation. This Helmert rotation, can increase the significance of the first estimated factor and reduce the significance of the remaining factors.[23]

McCulloch and Rossi (1990) have a very interesting paper where they use asymptotic principal components to extract the factors. They use over 2100 firms (NYSE and AMEX) from 1986 to 1987. This represents a significantly larger sample than most of the other papers reviewed. They focus on developing new tests for determining the adequacy of the model. Their primary test is a Bayesian estimation approach that looks at comparing posterior probabilities on the null (the APT approximate factor structure holds) and alternative hypothesis (the APT does not hold), based on the estimated factor loadings and prices. Their analysis shows

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[22]This result is analogous to Brown (1988) discussed in Chapter 2.

[23]The Helmert rotation is defined in Brown (1989). Brown suggests that the extracted factors could be transformed via an inverse Helmert rotation to obtain the true factors.

departures from the model restrictions. However, due to the high level of parameter uncertainty (since both the loadings and prices are estimated), they are unable to affirm or reject the APT. They also mention that common rules for determining k, such as looking for clear breakpoints in the eigenvalues, may not work. They find that the first eigenvalue is significantly larger than the others, but after that they remain fairly close together. They arbitrarily looked at one, three, and five factor models and find at most three pervasive factors.

Another, and somewhat more appealing, test for determining k, is developed by Connor and Korajczyk (1993). They agree with McCulloch and Rossi (1990) in that it is difficult to determine k based on the behavior of the eigenvalues. They note that standard likelihood ratio tests will not work unless a strict factor structure is assumed. When they are used with approximate factor structures they show that the bias will be positive (the tests will tend to identify too many factors). They develop a statistic that is based on the fact that if the correct k is chosen, there should be no significant decrease in the cross-sectional mean square of idiosyncratic returns in moving from k to $k + 1$ factors.

Using their newly developed test for significance, Connor and Korajczyk determine that from one to six factors is necessary. The first factor explains most of the variation but the second through sixth factors are significant and explain much of the variation associated with returns in the month of January. They therefore argue for a three to six factor model. As with their previous paper, they use simulation data to show that, in a world with four equally important factors: 1) their test for the number of factors works fairly well, and 2) the extracted factors are equal to the true factors (up to a non-singular transformation). This is important, in light of the results of Trzcinka (1986) and Brown (1989) discussed above.

3.3.3 Summary

One of the main tests of this dissertation will be whether or not security returns are generated by *k* economy wide, or pervasive, factors. Connor and Korajczyk's (1986, 1988) asymptotic principal components technique has several advantages over factor analysis in conducting such a test. First, it allows one to use a large number of securities, and diversification is critical in any test of the APT. Secondly, simulation has shown that it is possible to determine if the extracted factors are equal to the true factors.

Connor and Korajczyk's asymptotic principal components technique will be used to compare factors extracted from mutually exclusive portfolios of NYSE/AMEX/Nasdaq stocks. Unfortunately, this method is not well suited for comparing the prices of the extracted factors. First, the factors are still subject to a linear transformation, making it impossible to compare the price of the first factor from one sample to the price of the first factor from another sample. Second, the traditional Fama-MacBeth approach assumes a strict factor structure in the second stage, cross-sectional regressions. These issues will be addressed in the following sections.

3.4 Pre-Specified Factors

As discussed in Chapter 2, one intuitive way to tackle the empirical problem of factor identification is to simply pre-specify the factors.[24] Economic theory offers insight as to what factors might systematically affect the return on a security and the plethora of recorded economic variables offer observable proxies for these factors. Although this technique is nice in that we can obtain time series data on the

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[24]In fact, this is the only way to "identify" the factors (and then we are only identifying proxies for the factors). All of the other techniques mentioned earlier extract, but do not identify, the factors from the data.

underlying factors it is also dangerous in that one must arbitrarily choose which variables to use. As previously discussed, it also opens the door to a host of econometric issues. However, pre-specified factors allow for individual *t*-tests for significance. These t-tests can be run for various sub-samples and directly compared to determine if the price is the same. For this reason, they will be used to provide a cleaner test of the APT's pricing relationship.

3.4.1 Initial Results

Chen, Roll, and Ross (CRR) (1986) is considered the seminal work in this area. Several studies have relied on their basic methodology. This section will highlight some of the major results. The following section will look at various refinements and advances to their technique. CRR propose a list of several relevant macroeconomic factors and then identify observable proxies for each of them. Each of the proxies has to measure the unanticipated movement (or innovation) of the relevant factors. If one simply used the underlying variable, without accounting for expected movement, it would result in an errors-in-variables problem, thus rendering the usual statistical tests useless.

CRR point out that there are two logical approaches for obtaining the innovations. First, since many monthly rate of return observations and changes in growth rates generally show little serial correlation, they could be used directly as innovations. Alternatively, one could specify an equation (e.g. an autoregressive model) for expected movement and use the residuals as the innovations. They suggest that there is a tradeoff between the errors caused by using monthly growth rates (which might not have completely filtered out the expected movement) and the error introduced by incorrectly specifying the expected movement equation if one tries the alternative approach. CRR choose the somewhat simpler approach of using changes in growth rates or actual growth rates as the innovations.

CRR look at the following macroeconomic variables as proxies for the underlying factors: industrial production (monthly and yearly), inflation (both unanticipated inflation and changes in expected inflation), a risk premium variable (as measured by the delta between low-grade corporate bonds and long-term government bonds), and a term structure variable (as measured by the delta between long term government bonds and one month treasury bills). They eventually drop the yearly industrial production variable due to its high correlation with monthly industrial production. They also look at both the CRSP value and equally weighted market index, real consumption, and oil prices.

As with most of the other papers discussed in this review, a variant of the Fama-MacBeth approach is used in their study. CRR first run time series regressions of the asset's returns on the state variables to estimate the factor loadings (or beta values). They then run cross-sectional regressions of asset returns on the estimated factor loadings. The cross-sectional regressions were run month by month for several years to obtain a time-series of the associated risk premiums for each state variable or factor. The time-series means are then tested by a *t*-test for significant differences from zero. This procedure forces a strict factor structure on the returns in assuming that the residuals are uncorrelated.

CRR group the securities into portfolios to partially alleviate the errors-in-variables problem caused by using estimated factor loadings in the cross-sectional regressions. In order to obtain good dispersion for the estimated beta values, CRR grouped the portfolios based on size, since size is known to be related to returns.[25]

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[25]CRR also looked at various other schemes (e.g., stock price, betas on a market index, etc.) for grouping the securities into portfolios. They found that the grouping played a large role in which factors were priced. This is discouraging and offers a good area for future research.

CRR first look only at the macroeconomic variables and find that the following have significant risk premiums (for the entire sample period), where the sign indicates whether the risk premium is positive or negative: monthly industrial production (+), risk premium (+), and unanticipated inflation (-). They also find weak significance of expected inflation (-), and term structure (-). As one might expect, there were differences when the period was broken into sub-samples. For example, the two inflation related factors were highly significant between 1968 and 1977 (a period of high inflation) and insignificant in all other periods.

When they added in the market indices (both value and equally weighted), neither was deemed significant, although without the other factors they were both individually found to be significant. This seems to reject the CAPM model in favor of the APT. The proxies for consumption and oil prices were never significant. CRR thus conclude that stock returns are exposed to multiple systematic factors that affect their returns and that proxies for these factors can be found using simple financial and macroeconomic theory.

Hamao (1988, 1992) performs an empirical study similar to CRR on the Japanese stock market. He finds that expected inflation, term structure, and a risk premium proxy are significantly priced. Monthly production, and a trade term proxy are weakly significant. Changes in the exchange rate and oil prices are not significantly priced. Interestingly, he finds that the inflation proxy has a positive risk premium, whereas, CRR found a negative risk premium.

Chan, Chen, and Hsieh (CCH) (1985) also use pre-specified factors in a paper that examines the firm size effect. Their paper looked at NYSE stocks from 1953 to 1977. CCH use the same macroeconomic variables, and methodology, as CRR in the pricing equation. They find that their APT model does capture the size effect. Interestingly, they find the same significant factors as CRR. This is encouraging considering that each paper is based on overlapping but different sample periods.

Unfortunately, both papers look at the same macroeconomic variables. So even though they both support the APT we don't know if they have identified all of the appropriate factors.

3.4.2 Econometric Refinements

There is an extensive volume of literature that looks at pre-specified factors in the context of an APT. The following paragraphs will review some of the major contributions and advancements in this area.

Clare and Thomas (1994), using data from the London stock exchange, provide another important work in this area. Clare and Thomas start by looking at 19 proposed variables, they estimate the risk premiums and associated t-values and then eliminate the most insignificant. They then reiterate the entire process until they have found a model in which every risk premium is significant at the 10% level. They consider the resulting model to be the most parsimonious available, given their list of variables.

Clare and Thomas (1994) believe that CRR's approach does not truly give the innovation. They point out that CRR's innovations are still autocorrelated and therefore not innovations. Clare and Thomas use autoregressive models to provide the innovations.

Clare and Thomas group securities into portfolios using two methods (beta and size rankings).[26] When ordering by market beta, they find seven priced factors priced (although two of them appear to be only weakly significant). When they sort by market size (as CRR and CCH did) they find only two priced factors. This confirms the sensitivity of results to the grouping scheme as reported in CRR and also lends support to the hypothesis that some of the factors are sample specific, and not

[26]The astute reader will remember that CRR discussed the sensitivity of their results to the portfolio grouping scheme, but only presented results from one case (market size). Clare and Thomas present results from both schemes.

pervasive factors. Interestingly, unlike when they sorted by market beta, the market index does have explanatory power when added to this model.

Clare and Thomas's results are similar to those of CRR and CCH with two primary exceptions: 1) oil prices were not significant in the other papers, and 2) they find a positive premium on inflation. They suggest that these results are due to the level of "net" oil exports in the UK and the apparent fact that UK stocks were not a hedge against inflation during the period studied.

So far, most of the papers discussed in this chapter have relied on the Fama and MacBeth two step methodology. As previously mentioned, this methodology involves an errors-in-variables (EIV) problem. CRR and many others assume that the EIV problem is virtually eliminated by grouping the securities into portfolios during the second stage regression. However, Shanken (1992b) shows that after a correction is made to the *t*-statistic for the EIV problem, none of the original CRR factors are significant. Clare, Priestly, and Thomas (1997) make this same correction to the data in their original study, discussed above, and also find that many of the factors become insignificant. Shanken (1992b) also suggests restrictions on those proxies specified as differences in returns (since they also should satisfy the pricing relationship).

Several techniques, have been developed to overcome this EIV problem. McElroy and Burmeister (1988) develop a technique based on iterated nonlinear seemingly unrelated regressions (ITNLSUR).[27] Using macroeconomic proxies, similar to those used by CRR, they test a linear factor model that admits the APT and the CAPM as special cases. By estimating the factors and their associated prices simultaneously, this technique avoids the EIV problem. It also assumes an approximate factor structure. Their evidence is supportive of the APT. Brown and Otsuki (1992) employ the same methodology in a study using Japanese securities and

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[27]McElroy and Burmeister also provide an excellent discussion of the econometric difficulties associated with the Fama-MacBeth approach.

an expanded array of macroeconomic variables. They find evidence of five or six priced factors. The primary disadvantage of the ITNLSUR technique is that it is computationally burdensome and allows only a small number of securities to be used relative to asymptotic principal components.

Priestley (1996) argues that both the rate of change and autoregressive approaches to obtaining unanticipated components of macroeconomic variables are flawed. Specifically, he shows that the rate of change methodology (used by CRR (1986) and others) provides autocorrelated unexpected components; and the autoregressive methodology (used by Clare and Thomas (1994)) allows agents to make systematic forecast errors. Priestley then shows that the estimated risk premia are sensitive to the measurement of the unanticipated components. He finds that innovations estimated via a Kalman filter, do not violate the above conditions, and provide a better description of expected returns, both in and out of sample.[28]

Kramer (1994) uses a multi-factor APT model to study the role of macroeconomic factors in explaining the January effect. The empirical significance of January for stock returns is highly documented in the literature. Kramer points out that any explanation of this phenomenon must specify some seasonality in expected returns. Since there is a pervasive sense of seasonality in the economy, macroeconomic variables are a good choice.

Kramer looks at a five factor model using proxies for default risk, maturity risk, inflation, consumption growth, and a stock market proxy.[29] Kramer seasonally adjusts the inflation and consumption factors to decrease the chance that the factor

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[28]McElroy and Burmeister (1988) and Burmeister and McElroy (1988) also used Kalman filter techniques.

[29]The stock market proxy is a time series of residuals of a regression of the equally weighted CRSP index on the other four factors. So, in essence the model yields the CAPM as a special case.

mimics the anomaly and therefore yields a false positive statistic. Kramer uses the standard Fama-MacBeth method, but makes an adjustment in calculating the standard errors to compensate for the errors in variables problem. He concludes that the multifactor APT outperforms the standard CAPM and finds no evidence for a January effect in excess returns from his multifactor model.

Elton, Gruber, and Blake (1995) [EGB] look at an APT model for explaining returns in the bond market. They use bond indexes as well as macroeconomic variables as proxies for the factors. As always, the models require using unanticipated changes or innovations in the economic variables. One of the unique, and very appealing, contributions of this paper is the use of publicly available forecasts to measure unexpected changes in expectations for some of their proxies. This expectational data is probably a closer proxy for the factors that actually drive returns. Unfortunately, this data is somewhat scarce and generally only available for later time periods.

EGB look at a seven factor model and find that the two factors based on expectational data (unanticipated changes in inflation and gross national product) explain a large portion of the return variance and are significant at the 1% level. Another interesting point in EGB's study is empirical evidence that the market return indices are the most important variables in explaining the time series of returns but the macroeconomic variables lead to a large improvement when looking at the cross-section of expected returns.

Gangopadhyay (1996) uses a model similar to CRR to show that the seasonal mean reversion in stock portfolio returns is related to the macroeconomic proxies. The explained portion of the returns exhibit January mean reversion and the unexplained returns do not.

Most of the previous papers have looked at ways to improve the proxies or methodology that CRR used. Other researchers have explored new areas for finding

proxies. Young, Berry, Harvey, and Page (1987, 1991) look at the ability of financial statement variables to forecast betas in an APT sense. Booth and Booth (1997) look at the impact of monetary policy on security returns. Specifically, they find that the federal funds rate and an index based on the change in the discount rate are significantly priced. Jorion (1991) provides weak evidence that exchange rate risk is not a priced factor in an APT setting.[30] Conover (1997) however uses a larger number of exchange rates and foreign interest rates and finds several are priced. Chow, Lee, and Solt (1997) also find evidence of priced exchange rate risk within the context of a Fama-French (1993) model.

3.4.3 Summary

Numerous papers have shown that there is a direct link between various macroeconomic variables and the expected returns of securities. By using these variables as proxies for the underlying factors we assign some economic meaning to the APT. Many researchers advocate these techniques because they avoid the econometric difficulties associated with extracting factors from the data. Unfortunately, they often overlook the fact that the use of these variables incorporates even more econometric and theoretical challenges (e.g., multicollinearity, arbitrary choice of variables, timing issues, computing actual innovations, etc.). However, this technique allows one to estimate risk premia using a large number of securities and then directly compare the risk premiums estimated from different portfolios. As such, this technique will be used to test the stability of the risk premiums.

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[30]Many recent papers have documented relationships between stock prices and exchange rates. See for example Ajayi and Mougue (1996) and Bartov, Bodnar, and Kaul (1996).

3.5 Comparing the Various Techniques

Asymptotic principal components appears to offer econometric advantages over factor analysis. Macroeconomic proxies offer intuitive appeal and allow one to compare risk premia across groups. Numerous researchers have compared the various techniques from both a theoretical and empirical standpoint. This section will briefly review the important findings.

From a theoretical standpoint, the primary difference among the various techniques is the assumption of either a strict or approximate factor structure. Factor analysis assumes a strict factor structure, whereas asymptotic principal components assumes an approximate factor structure. The Fama-MacBeth two step approach, as used in most of the macroeconomic factor models, also assumes a strict factor structure in the second step cross-sectional regressions.

Burmeister and McElroy (1988) look at the empirical implications of the assumed factor structure using a model with both pre-specified and unobservable factors.[31] They develop two new multivariate approaches to complement their ITNLSUR approach, developed in McElroy and Burmeister (1988) and discussed above. One of these techniques, iterated nonlinear weighted least squares (INLSLS), is analogous to INLSUR with the notable exception that it allows one to force a strict factor structure. They find similar results for these two techniques and argue that, at least for the set of factors they used, the choice of factor structure is insignificant.

Burmeister and McElroy (1988) make another valuable contribution to the literature in this paper. They point out that in empirical asset-pricing studies it is common to assume zero residual risk on portfolios used to represent factors (mimicking portfolios) and therefore these portfolios can be used as exogenous variables in estimating risk premiums. Their third technique, iterated nonlinear three

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[31]They use a mimicking portfolio technique for the unobservable factors.

stage least squares (ITNL3SLS), allows for non-zero residual risk. They find that this model provides different results from either of the other techniques.

Huang and Jo (1992), building on Grinblatt and Titman's (1985) theoretical paper, demonstrate that a strict factor structure and an approximate factor structure coverage to one another as the number of securities is increased. Specifically they show that as the number of assets increase, an appropriately rotated matrix of factor loadings estimated from factor analysis converges to the matrix of eigenvalues from principal components.

Garrett and Priestley (1997), using the techniques of Burmeister and McElroy (1988), investigate the factor structure in relation to returns on securities from the London Stock Exchange. They argue that these returns are best described by an approximate factor structure and they find six or seven priced factors. More importantly, they argue that Shanken's (1992) critique of Chen, Roll, and Ross's (1980) results is not evidence against the APT, but rather evidence that the wrong assumption was made regarding the idiosyncratic covariance matrix. Clare, Priestley, and Thomas (1997) reinforce these findings by reexamining the same data used in Clare and Thomas (1994). When recalculating their results with Shanken's EIV correction or using Burmeister and McElroy's ITNL3SLS approach that assumes a strict factor structure, but does not suffer from the EIV problem, they find no significant factors. When they allow for an approximate factor structure, they find five priced factors.[32]

Shukla and Trzcinka (1990) compare the technique of asymptotic principal components with that of factor analysis. Their motivation is concern that the two are equivalent only if the idiosyncratic risks of all firms are equal (which one reasonably assumes is not the case). Factor analysis is less constrained than principal

[32]The authors point out that these findings should also apply to tests of the CAPM and might shed some light on the empirical debate on the "death of Beta".

components because it estimates the idiosyncratic risks at the same time it estimates the factors. Principal components ignores the idiosyncratic risks. Therefore, Shukla and Trzcinka posit that factor analysis might outperform principal components. They find that a five factor model based on principal components does at least as well as the five factor model based on factor analysis in terms of R-squared values and closeness of the intercept term to zero. They find that either model outperforms a one factor CAPM model. Their five factor model was able to explain almost 40% of the variation in excess returns.

Huang and Jo (1995) look at whether the number of factors tends to vary with different data frequencies. Monthly, weekly, and daily returns are used to extract factors using both asymptotic principal components and factor analysis. They find that, after correcting the daily data for problems due to nonsychronous trading, the number of factors is stable across all frequencies. The results find only one or two significant factors.

Another popular line of research is aimed at comparing or relating the extracted factors to macroeconomic proxies. In my opinion, if the APT holds, this is a very valuable line of research. In essence, by extracting the factors we avoid having to make arbitrary choices about which macroeconomic variables to use, yet we give the extracted factors an intuitive meaning by relating them to economic variables.

One of the first papers to attempt to provide a link between extracted factors and other variables was Fogler, John, and Tipton (1981). They argue that if the CAPM held in some economy, one could still extract factors from a sample of 100 securities and find a second, significant factor that might be due to random shocks on several stocks. If the factors could be shown to consistently be linear combinations of economic variables, then it would make a strong case for being a systematic factor.

Their empirical analysis is crude and attempts to relate three extracted factors to a broad stock market index, a U.S. Treasury bill index, and a utility stock index.

Connor and Korajczyk (1988) provide some comparisons between their derived factors and some of the economic factors used by Chen, Roll, and Ross (1986). They regress the excess returns of junk bonds (JBRET) and long term government bonds (UTS) on their factor estimates. The two variables (JBRET and UTS) are related to the risk premium and term structure variables used by CRR. Connor and Korajczyk find that their first factor explains 7% to 40% of the variance in JBRET. With five factors it explains up to 59% of the variance. For UTS their first factor explains 0% to 11% and with five factors up to 49% of the variance. The high correlation suggests that the two sets of factors are related.

Kim and Wu (1987) take a somewhat different approach in relating macroeconomic variables to APT factors. First they obtain time-series data on approximately 15 macroeconomic variables. Next, they extract factors directly from the macroeconomic data and, using a PROMAX rotation, identify the most significant variables in each factor.[33] Kim and Wu find that three factors account for 88% of the variation in all economic variables. They then use these extracted factors in an APT setting. Taking this methodology one step further, Cho and Pak (1991) use interbattery factory analysis to extract 13 factors that are common between a group of security returns and a group of macroeconomic variables. Starting with 98 macroeconomic variables they are able to rotate the initial factor loadings and obtain macroeconomic interpretations on 10 out of the 13 factors. In a recent paper, Zhou (1996) develops a generalized method of moments (GMM) technique that estimates what linear combination of economic variables is best able to forecast unobservable factors.

[33]A PROMAX roation seeks to rotate the factors in a manner that each one contains only a few highly loaded variables - thus making it easier to interpret them.

3.6 Various Other Techniques and Contributions

Their are numerous other techniques for estimating and testing the APT that can be found in the literature. Most of these techniques rely on advanced econometric techniques. For completeness, this section will briefly review some of the more prominent areas.

Mei (1993a, 1993b) develops a semiautoregressive technique (SAR). The intuition behind this technique is that past returns can be used as proxies for the factor loadings because they span the same return space as the betas. In this sense the technique is similar to the technique of substituting mimicking factor portfolio returns for unobservable factors. One major advantage of this technique is that it provides an asymptotic covariance matrix for the factor estimates.

Mukherjee and Naka (1995) apply a six factor model to the Japanese stock market. They employ a vector error correction which has advantages in its ability to explore dynamic co-movements among the variables examined even with nonstationary data. Mukherjee and Naka choose six macroeconomic factors and simply use the first difference of natural logs to approximate the innovations. For the period 1971 to 1990 they find that all of the variables are cointegrated with the market returns.

Bansal and Viswanathan (1993) argue that the main theoretical and empirical result of the APT is the existence of a nonnegative pricing kernel. They suggest that the linear relationship between the factors and the prices is overly restrictive. They develop a GMM approach to estimate the pricing kernel and find that their results reject a linear APT but support a nonlinear model. In a companion paper, Bansal, Hsieh, and Viswanathan (1993) apply the methodology to international data and find further evidence supporting their nonlinear model. Similarly, Abken, Jarrow, and Madan (1996) also propose a nonlinear APT and use options data to test the validity of the APT.

These nonlinear APT models are similar in nature to conditional APTs. Numerous papers have looked at conditional APT models which allow for time varying betas and risk premiums. Examples include: Engle, Ng, and Rothschild (1990), Koutoulas and Kryzanowski (1996), He, Kan, Ng, and Zhang (1996), and Kryzanowski, Lalancette and To (1997). The econometrics for these models are increasingly complex and the evidence is mixed. In a recent paper, Ghysels (1997) argue that the dynamics affecting time varying betas are not well understood and if they are misspecified the pricing errors may be larger than in a constant beta models. They give empirical examples where the pricing error is indeed larger.

Finally, several recent papers have developed sophisticated new techniques for testing asset pricing models. Geweke and Zhou (1996) develop a Bayesian framework for measuring APT pricing deviations. Epps and Kramer (1996) develop a test that is based on how the cross-sectional variance of returns should depend over time on the factor realizations. Velu and Zhou (1996) propose a GMM technique for testing multifactor models that is valid under very minimual assumptions. Interestingly, all of these papers reject the APT in favor of the CAPM.

3.7 Summary

The literature on the APT is mixed. The APT appears to explain more of the cross-sectional variation in asset returns and it explains some of the anomalies associated with the CAPM. However there is still much uncertainty regarding the number and nature of the factors. While there has been some evidence that the factors might be sample specific, there are no conclusive results. Previous simulation results have shown that Connor and Korajczyk's asymptotic principal components technique is an effective method for extracting factors. I hope to show that their technique can also be used to directly compare factors extracted from different portfolios. This concludes the review of the current literature. The following chapter

will develop the testable hypotheses and methodology, reviewing previous work as necessary.

CHAPTER 4

TESTABLE HYPOTHESES AND METHODOLOGY

This chapter develops my testable hypotheses and the methodology I will use to test them. First, I provide a brief review of the empirical specification.

This empirical specification of the APT closely follows Connor and Korajczyk (1988) and relies on Connor's (1984) Equilibrium version of the APT. As discussed in Chapter 2 (and presented again for ease of the reader) this version of the APT assumes that the random returns of securities are generated by a linear k factor model of the following form.

$$\begin{aligned} \tilde{r}_t &= E_t + B\tilde{f}_t + \tilde{\epsilon}_t \\ E[\tilde{f}_t] &= 0 \\ E[\tilde{\epsilon}_t | f_t] &= 0 \\ E[\tilde{\epsilon}_t \tilde{\epsilon}_t'] &= \Sigma_R \end{aligned} \tag{4.1}$$

where E_t is an n-vector of expected returns, B is a n x k matrix of the factor loadings, $\tilde{f}_t$ is a k-vector of pervasive factors, and $\tilde{\epsilon}_t$ is a n-vector of idiosyncratic returns. I assume an approximate factor structure so that Σ_R is not constrained to be diagonal.

If a risk-free rate exists and random returns follow a model as in equation (4.1), the equilibrium version of the APT gives us:

$$\begin{aligned} E_t &= \lambda_0 \, B \, \lambda_t \\ &= r_{f_t} 1_n + B\lambda_t \end{aligned} \tag{4.2}$$

where r_{f_t} is the risk free rate, E_t is the expected return, 1_n is a n-vector of ones, and λ_t is a k-vector of the risk premiums (or price of the risk). Combining equations (4.1) and (4.2) yields:

$$\tilde{r}_t - r_{f_t} 1_n = B(\tilde{f}_t + \lambda_t) = \tilde{\varepsilon}_t \tag{4.3}$$

with a time-series of T return observations for each of the n securities, equation (4.3) is written as

$$R^n = B^n F + \varepsilon^n \tag{4.4}$$

where R^n is the n x T matrix of asset excess returns (returns in excess of the risk free rate), B is a n x k matrix of factor loadings, F is the k x T matrix of $(\tilde{f}_t + \lambda_t)$ values, and ε^n is a n x T matrix of realized idiosyncratic error terms.

4.1 Testable Hypotheses

The testable hypotheses come directly from the above equations. Tests of the pricing implication of the APT equation (4.2) are subject to several criticisms. Primarily, with the equilibrium version, any test of equation 4.2 is a joint test that some portfolio is mean-variance efficient relative to the assumed factor structure. This statement is somewhat less critical than the similar critique of the CAPM provided by Roll (1977) which states that a specific portfolio (the market portfolio) is globally mean-variance efficient. Also, with an approximate factor structure, statistical tests based on a factors price are very difficult. In general one needs to invert a sample's covariance matrix to compute the appropriate test statistic. This task is computationally burdensome for very large portfolios and unfeasible for the normal case where the number of assets is greater than the number of time periods.

Tests of the linear factor generating structure avoid some of these difficulties. If the factor structure is rejected then it is not necessary to test the pricing relationship as it will not hold. If the assumption can not be rejected, test of the pricing implications are necessary.

The expected return of each asset is an unknown parameter, therefore, it is impossible to directly estimate the factors from equation (4.1). One could use the time-series mean of the returns as an estimate but this will introduce a bias of an unknown magnitude and direction. Equation (4.4) allows one to extract the time-series of the factor plus its associated risk premium (F in equation (4.4)). Unfortunately it is impossible to separate these two parameters and the test becomes a joint test of the equality of the factors and their associated risk premium or price.

The equilibrium expected return relationship in equation (4.4) reveals that the price of a factor is not dependent on the underlying security (i.e., the price is associated with the factor not the security). If a factor is pervasive it affects all assets and the price of the factor is the same for all assets. Therefore, if I draw two random samples of asset's from the same population and time period the matrix F in equation (4.4) will be the same. More specifically, equation (4.4) for sample one and two respectively is:

$$R_1^n = B_1^n F_1 + \varepsilon_1^n$$
$$R_2^n = B_2^n F_2 + \varepsilon_2^n \tag{4.5}$$

from which I obtain my first hypothesis:

<u>HYPOTHESIS #1</u>:

H_0: $F_1 = F_2$

H_a: $F_1 \neq F_2$

This hypothesis jointly tests the equality of the factors and the associated prices. I would like to test only the equality of factors but can not extract the factors independently of the prices. Therefore as a check for robustness I check the equality

of prices estimated from different samples. If the factor is pervasive the price will be the same. If, on the other hand, the factor is not pervasive and only affects one of the samples, the price need not be the same. The price of a factor can vary over time but by looking at the time-series mean from equation (4.2) I obtain my second hypothesis:

HYPOTHESIS #2:

$$H_0: \quad E(\tilde{\lambda}_t^1) = E(\tilde{\lambda}_t^2)$$

$$H_a: \quad E(\tilde{\lambda}_t^1) \neq E(\tilde{\lambda}_t^2)$$

where $\tilde{\lambda}_t^1$ *and* $\tilde{\lambda}_t^2$ are (k + 1) vectors of the first and second samples price's respectively. The first element is λ_0 and should be equal to the risk-free or zero beta rate.

4.2 Methodology

This section discusses my methodology for testing the hypotheses developed in the preceding section. I will also compare and contrast my methods with those of past researchers and highlight the unique contributions of this research.

4.2.1 Hypothesis #1 - Pervasiveness of Factors

The primary contribution of this research is the development of a methodology that allows for a direct comparison of factors extracted from separate groups of security returns. Very few researchers have looked at this issue.

Brown and Weinstein (1983) base their comparison on a set of factor's ability to explain the cross-sectional variation in returns. Their test does not compare each individual factor and therefore offers little insight as to whether the same factors are generating the returns. Cho (1984) uses inter-battery factor analysis and forces the factors to be common across groups. By analyzing the factors common to a large

number of groups (using two groups at a time), Cho finds that the number of factors between groups is stable. Again, however, there is no guarantee that it is the same set of factors that are common between each of the pairs. Conway and Reinganum (1988) use cross-validation techniques to determine if certain factors are sample specific. They find that only the first one or two factors extracted from a sample improve on the out-of-sample explanatory power. Conway and Reinganum, however, base their results on a very limited analysis.

My methodology is unique in several key areas. First, I will use sample sizes in the range of 1500 to 3000 securities. All of the papers discussed above used factor analysis and therefore were constrained to sample sizes of 30 to 60 securities. This is an important issue given the asymptotic nature of the APT. It is not surprising that in a small portfolio an idiosyncratic shock to one or two securities might be picked up as a significant factor. The likelihood of such an occurrence decreases as the sample size increases. Secondly, the previous comparisons assumed a strict factor structure, whereas I will assume an approximate factor structure. Garret and Priestley (1997), among others, provide evidence that security returns are best described by an approximate factor structure. More importantly, Garret and Priestly also show that empirical results are sensitive to the assumed factor structure (strict versus approximate).

My methodology is based on a regression technique developed in Connor and Korajczyk (1988, 1993). Connor and Korajczyk's asymptotic principal components technique extracts factors that are linear combinations of the true, but unknown, factors. With a monte-carlo evaluation of simulated returns data they demonstrate that the extracted factors can be compared to the true factors by looking at the R-squared values obtained in a regression of each extracted factor (one at a time) on the full set of true factors. I extend this methodology to compare extracted factors across sub-samples. I use extensive simulations to evaluate what the R-squared values will

look like under a variety of proposed factor structures. I then evaluate the R-squared values obtained from regressing factors extracted from real world data on factors extracted from a different sample of real world data. Based on the simulated data I can then make inferences about whether or not the evidence supports or rejects the first hypothesis.

As developed in Chapter 3, Connor and Korajczyk develop a technique for extracting the factors from the following *T x T* product matrix

$$\Omega^n = \left(\frac{1}{n}\right) R^{n'} R^n \tag{4.6}$$

Connor and Korajczyk then prove that

$$G^n = L^n F + \Phi^n \tag{4.7}$$

where G^n is an orthonormal $k \; x \; T$ matrix whose columns are the first k eigenvectors of Ω^n, L^n is a nonsingular matrix, and Φ^n converges in probability to the zero matrix.[34] I use the IMSL routine EVESF to extract the k largest eigenvalues and their associated orthonormal eigenvectors from the cross-product matrix in equation (4.6).[35]

The extracted factors can not be compared on a one-to-one basis with the true factors due to the rotational indeterminacy (as seen by L^n in equation (4.7)). However, Connor and Korajczyk (1988, 1993) show that equation (4.7) provides a method for evaluating the extraction technique. Equation (4.7) shows that each of the extracted factors is a linear combination of the true factors plus an error term (the Φ matrix) that approaches zero in the limit. So if one regresses each of the extracted

[34]Connor and Korajczyk provide a proof of this is in their 1986, 1988, and 1993 papers. Perhaps the clearest development can be found in Theorem 1 of their 1988 paper.

[35]With a few exceptions, all the data manipulation and testing is done in FORTRAN making extensive use of the IMSL (MATH and STAT) routines.

factors, one at a time, on the full set of true factors the R^2 values should approach one as n approaches infinity (this technique will be referred to as the factor regression technique). The R^2 value will deviate from one for small samples because of the error term. Connor and Korajczyk use simulated data to demonstrate the effectiveness of the technique for various factor structures.

I essentially repeat Connor and Korajczyk's analysis for my data sample with three primary differences: I look at various sample sizes; I expand on the number of factor structures analyzed; and I simulate the returns based on a k-factor structure and then extract $(k + 2)$ factors. These simulations will show that the extraction technique works and is robust to the underlying factor structure. The simulations also provide evidence on how large the samples need to be to effectively ignore the error term and show that the technique can distinguish "true" factors from false factors (the extra two factors that were extracted). Details of the analysis and each of the proposed factor structures are provided in my Chapter 6.

The most significant, and unique, contribution of my research is using equation (4.7) to compare sets of factors extracted from different samples. From equation (4.7), let

$$G_1^n = L_1^n F + \Phi_1^n \tag{4.8}$$

and

$$G_2^n = L_2^n F + \Phi_2^n \tag{4.9}$$

for samples one and two respectively. I assume that n is sufficiently large so that the error term in equation (4.8) is negligible (this assumption will be supported with evidence from simulations). Since L_2^n is nonsingular, I solve equation (4.8) for F and plug the result into equation (4.9) yielding:

$$\begin{aligned} G_1^n &= L_1^n ({L_2^n}^{-1} G_2^n) \\ &= L_*^n G_2^n \end{aligned} \tag{4.10}$$

Therefore, if the returns from the two samples are generated by the same *k* factors; the factors from sample one will be linear combinations of the factors from sample two. I can therefore use the regression technique described above to compare the two sets of factors.

For each assumed factor structure, I run a monte-carlo evaluation by extracting subsets of the simulated returns and comparing the two sets of factors using the regression technique discussed above. After evaluating the simulation data, I run monte-carlo evaluations based on the real sample data. I use a *t*-test to compare the mean R-squared values from the real data to the simulated data in order to identify those factors that are pervasive.

4.2.2 Hypothesis #2 - Equality of Risk Premiums

Numerous papers have compared risk premiums across different time periods. Very few have compared the risk premiums estimated from the two distinct samples over the same time period. Brown and Weinstein (1983) is one of the few exceptions. They compare the entire vector of risk premiums from extracted factors (since individual *t*-tests are not valid). Brown and Weinstein reject equality in every case.[36] I use macroeconomic variables as proxies for the factors and can compare the price for an individual factor across groups.

The first hypothesis involves comparing the entire time-series of F_1 and F_2. For the second hypothesis I make use of the fact that the expected value of the factors is zero, so the value of the time-series mean of F in equation (4.4) will approach the expected value of the risk premium.

Although there is a considerable amount of literature looking at advanced techniques for estimating the risk premiums associated with macroeconomic proxies, I

[36]Brown and Winstein suggest that the rejections might be due to the extremely large number of daily observations on each security. After making a correction for this they find weak evidence of equality.

will follow the Fama-MacBeth technique used by Chen, Roll, and Ross (1986) and discussed in Chapter 3. This technique allows a large number of securities to be analyzed and is still prevalent in the literature.

The methodology consist of two main steps.[37] In the first step, I run a separate times-series regression of the following form to estimate the betas (sensitivities) for each of *i* portfolios:

$$r_{i,t} = \alpha_i + \beta_{i,MP}MP_t + \beta_{i,URP}URP_t + \beta_{i,UTS}UTS_t + \beta_{i,UI}UI_t + \beta_{i,DEI}DEI_t + \varepsilon_{i,t} \tag{4.11}$$

For the regression in equation (4.11) I use five years of data prior to the start of the sample period.

In the second step, the estimated beta coefficients are used as independent variables in cross-sectional regressions of the following form

$$r_{i,t} = \lambda_0 + \lambda_{MP}\hat{\beta}_{i,MP} + \lambda_{URP}\hat{\beta}_{i,URP} + \lambda_{UTS}\hat{\beta}_{i,UTS} + \lambda_{UI}\hat{\beta}_{i,UI} + \lambda_{DEI}\hat{\beta}_{i,DEI} + \epsilon_i \tag{4.12}$$

A separate cross-sectional regression is run for each month of the year following the five year estimation period. This procedure is repeated for each year in the sample period, resulting in a time-series of 60 observations for each of the risk premiums. The times-series mean of each of the risk premiums can be tested by a *t*-test for significant departures from zero. I will also use a *t*-test to compare the risk premiums estimated from unique subsets of the data.

In order to partially alleviate the errors-in-variables problem that arises by using the estimated beta values in the second-step regressions, I group the securities into portfolios before running the second-step regressions. It is critical that the sorting scheme provides a good dispersion in expected returns. If each portfolio has similar expected returns, the cross-sectional regressions will have little power to determine the prices.

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[37]The actual proxies, and their definitions are discussed in Chapter 5.

I look at two common approaches for grouping the portfolios; size and industry. I want the groupings to eliminate idiosyncratic shocks, but maintain a wide dispersion in exposure to the systematic shocks (i.e., maintain a wide dispersion in beta's). Groupings based on industries is an obvious choice for eliminating idiosyncratic risks. Size is perhaps the most common technique for grouping and clearly provides a good dispersion in returns.

Previous research has documented that the price of the factors is affected by the portfolio grouping scheme. This apparent violation of the APT is probably due to an inherent weakness in the empirical methodology. The portfolios need to be formed in a manner that disperses the securities with respect to their sensitivity to the various factors. If there is only one factor (as in tests of the CAPM) this task is easy. With just one factor, sorting on expected returns will automatically disperse the single beta. On the other hand, sorting on expected returns may or may not disperse all of the betas in an APT context. Warga (1989) argues that different grouping schemes will maximize dispersion of assets' betas for some factors but will yield little dispersion in the assets' betas for other factors. Therefore these methods will give precise estimates for some of the risk premiums and imprecise for others.

CHAPTER 5

DATA SELECTION AND SOURCES

This chapter provides a review of the various data sources used in this dissertation. Where necessary I discuss the selection criteria and compare my choices to those prevalent in the literature.

5.1 Security Returns

I use common stocks traded on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and the Nasdaq market. Returns data for the securities are obtained from the Center for Research in Security Prices (CRSP) 1995 Stock File. I use only continuously-listed firms with no missing returns data, thereby eliminating any error associated with estimated returns. At first glance one might argue that the use of only continuously-listed firms will bias the results; since it is possible that omitting a particular set of firms will cause one to omit relevant factors. However, all subsets will be subject to the same omission and therefore will still be affected by the same factors.

The APT is not constrained to any particular return period, and previous empirical studies use daily, weekly, and monthly return data. Huang and Jo (1992) show that the data frequency has no effect on the number of significant factors. Therefore, the choice of data frequency should not affect the robustness of this study. For the first hypothesis, factor equivalence across samples, the results will hinge on the asymptotic equivalency of a sample's T x T product matrix. Therefore, from an estimation standpoint, I prefer a large number of time-series observations.

Unfortunately, the parameters being estimated (e.g., betas and risk premiums) may not be stable over long time periods. In fact, there is a large body of literature suggesting these parameters vary with time. Daily data would maximize the number of observations for a given time period. Unfortunately, it is well documented that daily data can bias estimates of variances (due to asynchronous trading) and means (due to the bid-ask spread). Monthly data partially alleviates these problems but restricts the number of observations.

Therefore, I compromise and construct weekly returns from the daily CRSP observations.[38] Weekly returns are calculated as the compounded return, including any dividends, from the last trading day of the previous week to the last trading day of the current week. Therefore, most weeks will have five trading days but some will have less due to market holidays. For the second hypothesis, price equivalence across samples, I use monthly returns. This is due to the fact that most of the macroeconomic variables are only available on a monthly basis. Again, I construct the monthly returns from the CRSP daily observations. However, with monthly returns I allow for missing daily observations.

The final consideration is the length of the estimated interval; which, as mentioned above, is a choice between stationarity and sample size. I conduct the tests over a ten year interval and, when possible, two five year subintervals. The separate five year periods are 1986 through 1990 and 1991 through 1995.[39] For each

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[38]This is the same approach used in many studies, including those by Lehmann and Modest (1988), Shukla and Trzcinka (1990), and McCulloch and Rossi (1990). However, some of these define the week on a Wednesday to Tuesday basis since there are fewer holidays on these days and to alleviate any day of the week effect.

[39]One of the simulation models requires that t (the number of observations) be greater than n (the number of securities); in this case I do not look at the five year intervals.

five year interval I retrieve 260 weekly and 60 monthly returns. The final number of securities in the various samples range from 1680 to 3308.

5.2 Macroeconomic Variables

The methodology requires the use of excess returns (returns in excess of the risk-free rate). There is some questions as to the validity of short term risk-free rates implied from T-bills.[40] For this reason, I construct weekly risk-free rates using daily observations of a three month T-bill rate. The St. Louis Federal Reserve Bank's Economic Data Base (FRED) provides a daily rate for the shortest term T-bill having at least three months to maturity. For each week I calculate the average of these daily rates and convert the average into a weekly rate.

For the second hypothesis, I use the monthly T-bill rate from Ibbotson Associates (hereafter referred to as IA).[41] IA use the CRSP Government Bond file to find the shortest term bill having at least one month to maturity. They price the bill on the last day of the previous and current trading month and calculate a total return based on these prices.

As previously discussed, the macroeconomic proxies will only be used in testing the second hypothesis, equivalence of risk premiums across groups. I use five variables that are similar to those used by Chen, Roll, and Ross (1986). The CRSP equally weighted index is also used (both simultaneously with the other variables and alone). The five proxies are derived from various basic series discussed in the following paragraphs.

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[40]Returns on short term T-bills are highly volatile due to their use in many hedging strategies. Many thanks to Dr. Jim Hilliard for pointing this out and for his insightful discussions regarding risk-free rates.

[41]All of the Ibbotson & Associate's data is taken from *"Stocks, Bonds, Bills, and Inflation - 1995 Yearbook,"* published by Ibbotson & Associates.

5.2.1 Basic Monthly Series

1. CPI = seasonally adjusted consumer price index from the Bureau of Labor Statistics. There is some question as to the use of seasonally versus nonseasonally adjusted variables and examples of both can be found in the literature. To the extent that I want unexpected movements the use of seasonally adjusted variables makes more sense.
2. LGB = total return on a long term government bond from IA. Using data from the Wall Street Journal, IA choose one bond with a term of approximately 20 years. They choose a bond with a current coupon and without special tax features, call privileges, etc. The total return is based on the flat price (average of bid and ask prices) plus any accrued coupon interest.
3. TB = one month T-bill rate from IA (discussed under risk free rates section above).
4. IP = seasonally adjusted industrial production index from the *Survey of Current Business*.

5.2.2 Derived Monthly Proxies

Based on the above basic series I construct five proxies. I use the same names as Chen, Roll, and Ross (1986) but some of the derivations are slightly different. The proxies are discussed in the following paragraphs:

1. MP = monthly growth in industrial production. MP(t) = ln[IP(t)/IP(t-1)]. As discussed by Chen, Roll, and Ross, IP(t) is a measurement that lags actual activity by at least part of a month, so this variable will lead the other factors by one month.
2. URP = a measure of the risk premium associated with default spreads. URP(t) = LCB(t) - LGB(t). Chen, Roll, and Ross use the return on low-grade corporate bonds obtained from IA. This variable was not

available and thus I use a slightly different variable. A large portion of the return on junk bonds can be thought of as a call option on the underlying equity of the firm. Therefore, URP is somewhat related to a market index. This might partially explain the high significance of URP in Chen, Roll, and Ross's results.

3. UTS = a measure of the risk associated with maturity spreads. $UTS(t) = LGB(t) - TB(t)$.

4. UI = a measure of unanticipated inflation. $UI(t) = I(t) - E[I(t)/I(t\text{-}1)]$. Actual inflation is defined as the monthly difference in the logarithm of CPI (i.e., $I(t) = \ln[CPI(t)/CPI(t\text{-}1)]$). Next, I define expected inflation, $E[I(t)/I(t\text{-}1)]$, as the fitted values from an ARIMA(1,1,1) time-series model for $I(t)$. Then, $UI(t)$ is defined as the residual or unexpected component of the $I(t)$ time-series. A review of the literature shows numerous methods for constructing the expected inflation series. Chen, Roll, and Ross use the results of Fama and Gibbons (1984) who back out expected inflation from Fisher's Equation.[42] Others have used Kalman filtering techniques or various ARIMA specifications.

5. DEI = change in expected inflation. $DEI(t) = E[I(t+1)/I(t)] - E[I(t)/I(t\text{-}1)]$.

As mentioned in Chapter 3, there are numerous method for selecting and calculating the proxies. These five are chosen as a starting point based on their prevalence in the literature.

[42]The Fisher equation relates risk free rates to expected real returns and expected inflation.

5.2.3 Properties of the Macroeconomic Proxies

The macroeconomic proxies are assumed to be, or at least approximate, mean-zero innovations or unexpected movement. As such, the time-series of these variables should display little autocorrelation and their means should be close to zero. Table 5-1 lists the mean, variance, and autocorrelations for each of the macroeconomic variables. With the possible exception of the MP series, none of the autocorrelation functions appear to be large and thus the series can be used as innovations. The mean values are all near zero and therefore I do not attempt to subtract off a running mean.

Table 5-2 shows the correlations across the proxies. URP and UTS are highly correlated since they both have LGB in their definition. I define a second risk premium (URP-2) to try and lessen the collinearity between URP and UTS. First, I calculate the monthly yield differential between a portfolio of Baa rated commercial bonds and a portfolio of long-term government bonds (both given in Moody's annual bond manuals). URP-2 is then the first difference of the natural log of the yield differential series. Since yields are inversely correlated with returns URP-2 should be negatively correlated with URP. Unfortunately URP-2 is still highly correlated with UTS.

Unlike some researchers, I do not find a high correlation between UI and DEI, suggesting that for this period of data the ARIMA(1,1,1) model does an adequate job of filtering out the unanticipated component of inflation. As Table 5-2 illustrates all of the variables are somewhat correlated and this should be kept in mind when interpreting the results. Koutoulas and Kryzanowski (1996) orthogonalize the factors by successively regressing them on each other. This approach eliminates the collinearity but involves arbitrary choices as to the order of orthogonalization.

5.3 Summary

This concludes the discussion on the various data sources and proxies used in this research. A word of caution is necessary regarding the macroeconomic proxies. There are numerous choices being made when using these factors. These choices involve the specification of the innovations, the alignment of the proxies with the returns, collinearity issues, etc. I have lightly touched on some of these choices in the preceding sections and in Chapter 3. Previous papers have shown that the significance of factors is very sensitive to these and other issues and as such the results presented in the next chapter apply only to the proxies as I have defined them. Clearly much work remains to be done in this area.

CHAPTER 6

RESULTS

This chapter presents the empirical analysis. The first section looks at hypothesis #1 (equivalence of factors) and the second section looks at hypothesis #2 (equality of prices). When possible the results are compared to those found in the literature.

6.1 Hypothesis #1 - Equivalence of Factors

Tests of the first hypothesis involve comparing real world data to simulated data.[43] Before presenting the results, it is necessary to discuss the various simulation models used in this analysis. All of the models assume a five factor generating process. The key differences are in the assumed factor structure (i.e., strict versus approximate) and in how the factors, betas, and idiosyncratic terms are generated. In general, I need to generate a matrix of excess asset returns that follow the linear factor structure of equation (4.4). The following section discusses how the five models accomplish this task. Table 6-1 provides a summary of the techniques.

6.1.1 Simulation Models

The first model is based on the simulation used in Connor and Korajczyk (1988). Five factors are extracted from the full sample of security returns. These are assumed to be the "true" factors. The "true" betas are obtained by ordinary least

[43]Throughout this chapter the term "real world data" means actual returns from NYSE/AMEX/Nasdaq securities and "simulated data" means returns generated from the models described in the following section.

squares (OLS) regressions of the excess security returns on the extracted factors. The idiosyncratic terms are assumed to be temporally independent but cross-sectionally correlated. Specifically, let $\hat{\sigma}_i^2$ be the variance of the residuals from the above OLS regressions. The idiosyncratic return for asset i is

$$\begin{aligned} \epsilon_1 &= \eta_1 \\ \epsilon_i &= \rho\,\epsilon_{i-1} + \eta_i, \qquad i = 2, \cdots, n \end{aligned} \tag{6.1}$$

where η_i is a random draw from a normal distribution with mean zero and variance s_i^2 - where s_i^2 is set so that $\mathrm{var}(\varepsilon_i) = \hat{\sigma}_i^2$.

If $\rho = 0$ the simulated returns follow a strict factor structure. If $0 < \rho < 1$, then every securities idiosyncratic term is correlated with one another (cross-sectionally). The B^nF matrix is constant for every iteration, only the ε^n matrix is simulated.

Since the simulated factors are based on the extracted factors they will not be equally-important factors. Figures 6.1 through 6.3 show a plot of the eigenvalues for the various samples. These figures clearly show that the first one or two factors are dominant and the others may not be significant. Of course the extracted factors may have loaded heavily on the first eigenvector - if the other factors are pervasive however, the regression technique described in Chapter 4 should still work.

The second model is based on a model in Connor and Korajczyk (1993). This model is different from the first in several ways. First, it assumes equally-important factors, and second it allows for the factors and betas to be random draws and thus vary with each simulated draw. Following the development in Connor and Korajczyk (1993), the model of excess returns is:

$$r_{it} = B_i f_t + \epsilon_{it} \qquad i = 1, \cdots, n; \qquad t = 1, \cdots, T$$
$$B_i \sim MVN\,(1^5, \sigma_b^2 I^5)$$
$$f_t \sim MVN\,(\pi I^5,\ \sigma_f^2\ I^5)$$
$$\epsilon_t \sim MVN\,(0^n,\ \sigma_e^2 I^n) \tag{6.2}$$

where 1^5 is a 5 x 1 vector of ones, ε^t is a n x 1 vector of idiosyncratic returns for period t and 0^n is an n x 1 vector of zeros. As seen in equation (6.2) the betas are i.i.d. with mean 1 and variance σ_b^2, the factors are i.i.d. with mean risk-premium π and variance σ_f^2, and the idiosyncratic terms are i.i.d with mean zero and variance σ_e^2.

There are four parameters in the model, σ_b^2, σ_f^2, σ_e^2, and π. Connor and Korajczyk suggest the following scheme for setting the parameters. σ_e^2 is given by the cross-sectional average mean-squared residual after extracting 5 factors and running an OLS regression as in the first model.[44] The remaining three parameters are tied to the average excess return to the CRSP equally weighted index (E $[r_{ew}]$), the variance of the equally weighted index (σ^2_{ew}) and the variance of the average asset in the sample (σ_i^2). Details of the procedure are found in Connor and Korajczyk (1993).

In summary then, the second model uses simulated values for B^n, F, and ε^n. The model uses sample and market observations to set the parameters and each of the factors play an equal role in generating returns. The second model assumes a strict factor structure.

The third model is identical to the second model in every way but one; the third model sets the simulation parameters using only sample data. Specifically, I construct an equally weighted portfolio of the securities in my sample and use this

[44]Actually Connor and Korajczyk only use four factors in their model. Their methodology was adjusted accordingly for my five factor model.

portfolio's mean and variance in place of the CRSP equally weighted index's mean and variance. This distinction may be important since the CRSP equally weighted index has almost twice the expected return of my sample's index.[45] The ability to extract factors will depend on the relative proportion of the return that is explainable (i.e. B^nF) versus the idiosyncratic portion (ε^n). Using sample data to set ε^n and market data to set B^nF will bias the method towards being able to successfully extract the factors if the market index has a higher expected return than the sample.

The fourth model makes one modification to the third model. Instead of equally important factors, I scale the betas so that each factor is responsible for varying degrees of the expected component of returns. Specifically, I simulate the factors, betas, and residuals as in the third model but then I multiply B_1 by 4, B_2 by 2, B_3 by 1, B_4 by 0.5 and B_5 by 0.25. This scaling preserves the proportion of returns that is explainable but it shifts the amount attributable to each individual factor.

The fifth model is perhaps the most important from a theoretical standpoint. The first model allows for either a strict factor model or a factor model where every securities idiosyncratic terms are correlated. The second through fourth models all assume a strict factor structure. The asymptotic principal components technique, however, is based on an approximate factor structure. The fifth model generates B^n and F the same as in the third model but then models the idosyncratic portion as a random draw from a multivariate normal distribution with mean 0^5 and a variance-covariance matrix based on the actual residuals from the OLS regressions (the other techniques used only the diagonal elements of this matrix). I use the IMSL routine

[45]This difference is probably driven by the exclusion of newly listed firms, firms involved in mergers, and thinly traded firms - all of which generally outperform the market.

RNMVN to generate the random draw of ε^n. This routine uses a Cholesky decomposition of the sample covariance matrix to generate the multinormal variates.

In order to estimate the *n x n* covariance matrix for the sample, *n* (the number of securities) has to be less than *T* (the number of observations) so I use only the full 10 year period (not 5 year intervals) when using this model.

6.1.2 Comparing Factors Extracted From Real Data

The first results compare the factors extracted from mutually exclusive portfolios of actual security returns. I randomly choose two portfolios of *n* securities (where *n* is equal to 50, 150, 500, and 1500) and extract seven factors from each portfolio.[46] I then regress the factors from the first sample (one at a time) on all seven of the factors extracted from the second sample. I repeat this procedure 100 times and then calculate the mean and standard deviation of the R^2 values for each of the seven factors. These results, along with the maximum and minimum for each R^2 value, are provided in Table 6-2.

I could also have regressed the factors from the second sample on the factors from the first sample - since I do not know which set of factors is the "true" set. If the factors were identical the R^2 values would be the same in either case. This technique would have doubled my sample size but may have introduced a bias of an unknown direction and magnitude. The choice of seven for the number of factors is somewhat arbitrary. A survey of the current literature reveals a somewhat polar distribution - some feel there are only one or two factors while others argue there are a very large number. There is a tradeoff between extracting too few or too many factors. I would like to extract all of the pervasive factors and one or two non-pervasive or "false" factors. This would allow for a robust test of the technique since the pervasive factors should have R^2 values near one while the "false" factors will

[46]For the overall 10 year period I use $n = 1000$ instead of 1500 (since there are only 2334 securities in this sample).

not. Unfortunately extracting too many factors is computationally burdensome and might bias the results (since each additional factor acts as a linearly independent regressor).

Several conclusions are apparent from Table 6-2. First, the results are very dependent on both sample size and sample period. Just as one would expect, as the sample size increases so do some of the R^2 values. The primary explanation is that the "non-pervasive" factors become less prevalent as the idiosyncratic risk is diversified away. With small samples, one security might experience a stream of large idiosyncratic shocks. It is possible that one of the extracted factors will be perfectly (or near perfectly) correlated with a particular securities time-series of returns. Some researchers have thrown out these so called "Heywood" cases. Throwing out spurious returns is dangerous - how does one judge what is or is not "normal" in the market?

Table 6-2 clearly illustrates the pitfalls associated with trying to identify factors with very small portfolios. The R^2 values are extremely small and there is no clear break-point in the values between the factors. In the 1986 through 1990 period the results seem to indicate a one factor model, as only the first R^2 value approaches one as the sample size is increased. In the overall ten year period, and to some extent in the 1991 through 1995 period, the R^2 values for the first two factors seem to dominate the others. These results are consistent with the eigenvalue plots in Figures 6-1 through 6-3.

One possible explanation for the results in Table 6-2 is that the extraction technique loads up the significance of the first factor. This is a potential problem and the following section addresses it, and other issues, in more detail.

6.1.3 Validation of Techniques

The data presented in the previous section was suggestive of only one or two "pervasive" factors. By looking at the R^2 values obtained with simulated data I will

be able to make statistical inferences about the equality of the real world and simulated factor structure. These inferences are only valid if the asymptotic principal components technique can extract the factors with little error and if the regression technique for comparing factors is able to distinguish between pervasive and "false" factors. The results in this section suggest that both of the techniques (factor extraction and comparison) are robust.[47] I also look at the sensitivity of the techniques to the sample size.

With the simulated returns data I have the luxury of knowing the "true" factors. I can therefore compare the extracted factors to the true factors. For each of my five models I generate the same number of security returns as there are securities in the various time periods (e.g., for 1991 through 1995 I generate a time-series of 260 excess return observations for 3,673 securities). I then extract seven factors from the simulated returns and regress them on the full set of true factors. I run 25 iterations for each of the models.[48] Table 6-3 provides the results of this analysis.

Table 6-3 clearly demonstrates the ability of both of the techniques. For the most part, in every time period and across the models, the R^2 values of the first five factors approach one and the two "false" factors have very low (almost zero) R^2 values. For the assumed model structures and sample sizes the error term in equation (4.7) appears to be negligible. The results also indicate that even if the extraction technique loads up the significance of the first factor the regression method is still able to identify the other pervasive factors. This important result should dispel any

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[47]Obviously the two techniques are inseparable - if the factor regression technique shows the factors are not the same it could be a violation of either the factor extraction technique or the factor regression technique.

[48]Since I generate returns for the full sample, the simulations and factor extraction are computationally burdensome. Relative to the real world data, the standard deviation of the R^2 values are very small so I feel comfortable with only 25 iterations for these results.

fears that the rotational indeterminancy problem will make the task of factor comparison impossible. The following paragraphs highlight some other aspects of Table 6-3.

It is apparent (from Panel A in Table 6-3) that until the correlation coefficient becomes very large (e.g., 0.9) the results are insensitive to the value of rho. Subsequent work therefore only looks at the case where rho is equal to 0.5. The differences in the results from the second and third model (Panels B and C respectively in Table 6-3) are also virtually non-existent. Therefore, I only use the third model in subsequent work. The fourth model (Panel D) shows that scaling the relative importance of the betas does impact the techniques ability to extract them. However, only the ability to extract the fifth factor which was scaled by 0.25 seems to suffer (and even then it's R^2 value is significantly larger than the sixth and seventh extracted factors).

Panel E indicates that the technique is not as efficient with data from the fifth model. At first, this seems unfortunate since this is the only model that generates returns following an approximate factor structure. There is one very plausible explanation for this result. The sample size for the other models ranged from 2,334 securities to 3,367 securities, whereas the fifth model is limited to 500 securities. As indicated in Table 6-2 size seems to be a key issue and model five's limited sample size may bias the results.

The next analysis further investigates the issue of size by looking at factors extracted from smaller portfolios of simulated returns. Specifically, I extract factors from simulated returns using portfolios of 50, 150, 500, and 1500 securities and then I regress the extracted factors on the true factors. Tables 6-4 through 6-7 provide the results. Again, there is overwhelming evidence that size is critical when comparing factors. When the sample size is 50 (Table 6-4) the results seem to indicate only one factor, even though the returns are generated by five factors. This offers much

insight into the mixed results of previous studies using portfolios of only 30 securities.[49]

Tables 6-4 through 6-9 highlight one of the key differences between the models. The first and fifth model allow each security to have a different mean value for it's simulated idiosyncratic return. Models three and four assume the idiosyncratic shocks are identically distributed. With unequal idiosyncratic variances (as one would expect with real data), sample size is more important because it is more likely that one random security might have unusually large idiosyncratic shocks that are picked out as a "false" pervasive factor. By comparing the results in Panel A of Table 6-6 (model one with 500 securities) with the previously discussed results for model five in Panel D of Table 6-2 (which also used only 500 securities) we see that the portfolio size probably played a large role in the somewhat weak results for model five.

With a sample size of 1,500 as in Table 6-7 the results are encouraging, but somewhat mixed. With model three's equally explanatory factors (and perhaps more importantly) equal idiosyncratic variances the technique is near perfect (as seen in Panel B). The R^2 values for the first five factors are almost one and the R^2 values for the "false factors" are no greater than 0.001. Even with the scaled factor's of model four the technique is fairly robust.[50]

6.1.4 Statistical Inferences

Now that I have demonstrated the ability of the techniques I can compare the R^2 values from the sample data to the R^2 values from the simulated data and make

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[49]With small portfolios, factor analysis may outperform principal components since factor analysis attempts to explain only common variation whereas principal components explains total variation.

[50]It is important to realize that the first factor now has 16 times the explanatory power of the fifth factor and as such the fifth factory may be virtually insignificant.

inferences regarding their equality. Specifically, if the actual returns are generated by a k-factor model that is of the same form as one of the five tested models, the R^2 values should be similar. Tables 6-8 through 6-11 present the results for sample sizes of 50, 150, 500, and 1500 securities.

I base the comparisons on one parametric test and one nonparametric test. The parametric test is Satterthwaite's approximate test of equality in means for two independent, normally distributed samples with unequal variances. The nonparametric test is the Wilcoxon rank sum test for equality in means. The Wilcoxon test assumes the samples are independent and from the same distribution (but the distribution need not be normal). I use the nonparametric test because a Shapiro-Wilk test indicates that the R^2 values are not from a normal distribution.

Table 6-8 shows that for a sample size of 50 the hypothesis of equal means is rejected for almost every factor and every model with both of the test statistics. The one possible exception is the results for the first model in Panel B. This is encouraging considering that the "true" factors in the first model are extracted from the real world data that they are now being compared to. In fact, the first model is almost like a control sample. Given the previous results on the importance of sample size, little additional information can be gleaned from these small sample results in Table 6-8.

As the sample size increases, it is obvious that the real returns were not generated by a model with five equally important factors and equal idiosyncratic variances. The t-statistics for comparing model three and four to the real world model for some factors are in excess of 100 - resulting in overwhelming rejections of the null hypothesis that the mean R^2 values are the same.

The results from model one offer the most insight as to the true nature of the factor structure of the real world data. The factors extracted from the real world data are used as the "true" factors in the first model. Looking at Panel B in Table 6-11,

an interesting pattern emerges for the R^2 values of the simulated data versus those from the real world data in Panel A. First, in this case (and in several others), I notice that the results are very sensitive to the time period. The 1986-1990 time period consistently has higher *t*-statistics. This might be due in part to the October, 1987 market crash. Looking at the other two periods, there is some evidence that the first factor's R^2 values are the same but I reject the null that factor's two through five are the same.[51] These results strongly suggest that the real world data is not generated by a five factor model. In fact the data suggests there are at most two pervasive factors. This is perhaps one of the most significant results.

Overall the results are encouraging. I have not identified, nor do I attempt to identify, the actual factor structure generating the real security returns. I have, however, provided a screening test for evaluating any proposed factor structure. After comparing the real world data to several alternative factor structures I conclude that none of the proposed structures produce returns consistent with the observable results. I also conclude that the number of pervasive factors in the economy, at least for the time periods in my study, appears to be less than five and more likely one or two.

6.2 Hypothesis #2 - Risk Premiums

This section presents the results of my second hypothesis which states the price of factors should be the same across samples. As previously mentioned, macroeconomic proxies offer an intuitive appeal to the APT by giving a meaning to the unknown factors. Unfortunately, as past research has documented, the number and nature of the priced factors is sensitive to numerous issues (e.g., how the data is grouped into portfolios, how the macroeconomic variables are aligned with the

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[51]For the 1986-1995 period there is weak evidence that factors two and three have the same R^2 values.

returns data, how the innovations are specified, what time period is studied, etc.). In short, the cost of the macroeconomic proxies intuitive appeal is the spurious nature of pricing tests that use them.

My results suggest a new technique for evaluating the various proxies than can complement normal tests for significance. I start with an overall sample of 1,680 securities and employ the same methodology as used by Chen, Roll, and Ross (1986) and discussed in Chapter 4. I estimate a time-series of 120 observations (ten years of monthly data) for each of the risk premiums based upon numerous models - the models differ only in the number of macroeconomic proxies used. I then divide the sample in half (so that there are 840 securities in each sample) and repeat the procedure. Results for several of the models are provided.

6.2.1 Sorting Schemes

A portfolio grouping of returns is used to partially eliminate the errors-in-variables problem associated with the use of estimated betas in the cross-sectional regressions. The portfolios will eliminate much of the idiosyncratic risks and should therefore provide cleaner estimates of the prices. As mentioned earlier, the grouping scheme is of critical importance and several researchers have shown that the number and nature of the priced factors is dependent on the grouping scheme. The groupings need to eliminate the idiosyncratic risk and provide a good dispersion of estimated betas for all of the factors. Unfortunately, different schemes will tend to offer varying degrees of dispersion for the different factors and can thus bias the results. If there is little cross-sectional variation the OLS regressions will have a hard time accurately predicting the price.

I choose two grouping schemes. The first variable I group on is size. Specifically, I use the prior year's end-of-year market capitalization as provided by CRSP. I rank the securities according to size and then form 112 portfolios of 15

securities for the overall sample and 56 portfolios of 15 securities for the sub-samples.

I then repeat the entire analysis grouping by industry. I looked at several schemes for grouping by industry. One obvious choice is to use 2 digit SIC values. Unfortunately this did not provide a large enough number of portfolios and the number of securities in each portfolio was bi-modal - roughly half of the industries contained over 100 securities and the other half contained very few securities. Ideally, I would like a large number of industries (so that I have enough data points for the cross-sectional regressions) and a large number of securities in each portfolio (to effectively diversify away the idiosyncratic risk). *U.S. Industrial Outlook* provides a breakdown of over 100 industries based on 4 digit SIC, but I felt with that many industries, the number of securities in each portfolio would be too small. I compromised between the two techniques and used a breakdown of 48 industries provided by Fama and French (1997). Appendix C provides a list of the industries, the SIC codes they contain, and the number of securities from my sample in each of the industries. The number in each portfolio can vary and ranges from 4 to 153. Over one-half of the industries have at least 25 securities and the average number in each portfolio is 35. For the sub-samples, I randomly divide the securities in each of the industries.

Table 6-12 provides some measures of how effective the two sorting schemes are at dispersing the betas for each of the proxies. The various panels in Table 6-12 differ in the number of proxies included in the model. I would like the standard deviation for the betas to be large. As suggested, the sorting schemes tend to disperse some betas more so than others. For example, in Panel A (which uses all six of the proxies as defined in Chapter 5) I notice that the standard deviation of the betas for MP, EW, and UTS are small relative to those for UI, DEI, and URP. The level of dispersion is also affected by which variables are in the model as evidenced

by the remaining panels. There does not appear to be any large systematic differences in dispersion between the two sorting schemes.

6.2.2 Statistical Inferences

The second hypothesis involves comparing the prices of risk estimated from two different samples. Before doing so, it is informative to look at the estimated price of risk using the entire sample. As documented by previous researchers, I find that the estimated prices and their significance are very unstable across time-periods and models. Across time periods the variation is attributed to the different prevailing economic conditions. The variation across models is due to the correlation between the individual proxies. As different proxies are added or subtracted from the model, the prices can drastically change. This is an unfortunate, but unavoidable, problem.[52]

Table 6-13 shows the estimated price of the various risk factors for the ten and five year periods. Panel A is for the model with all of the variables, Panel B eliminates EW, Panel C eliminates EW and UTS, and Panel D eliminates EW and URP. I eliminate the market index since it should have no special role in the APT and I eliminate the risk premium and term structure variables (one at a time) because of their high correlation with each other.

There are several results worth highlighting. First, across all of the models and time periods, most of the factors are insignificantly priced and the intercept (which should be zero) is significantly priced. The few exceptions include UI, URP, and UTS which are priced in at least one case. Chen, Roll, and Ross (1986) found no significantly priced factors in their 1978-1984 time period (which is the closest time-period to my study). I also ran a model with just the equally weighted index

[52]Some argue this problem can be avoided by orthogonalizing the factors using progressive OLS regressions (e.g., regress factor two on factor one and use the residuals as the new factor two, then regress factor three on factors one and two, etc.). The problem is still there since the prices will depend on the order of orthogonalization.

and find that the price is negative but insignificant. This result supports Fama and French (1992) who find similar results using data from recent time periods. When I break the samples in half this pattern continues - different factors are priced in differing samples.

The sorting schemes also have an effect on the various prices. This is an interesting result given the somewhat consistent level of dispersion in betas across the two sorting schemes. I make no conjectures about the theories regarding the signs of the various risk premiums. They have no bearing on my research and it is somewhat frivolous to discuss the signs since most were insignificantly different than zero.

Table 6-13 highlights the already mentioned fundamental weakness of the macroeconomic proxies - they are simply not stable and as such any inferences should be made with caution. Of course the results in my study are only valid for my definitions of the various proxies.

At first, the results of the previous paragraphs are disheartening, why do I care if the risk premium is the same across samples if it is insignificant in the first place? The answer is simple; this technique may help screen out those factors that are significantly priced in some time-periods and not in others. Table 6-14 provides the results for this analysis. I conduct both a parametric and a nonparametric test. Unlike the independent samples of R^2 values used in testing hypothesis one, the two time-series of price estimates from the sub-samples are naturally paired (since they are estimated for the same month). Therefore, I use a paired *t*-test and a Wilcoxon signed rank test for the second hypothesis.

The results are given in Table 6-14. As one might predict, it is hard to reject the equality of the risk premiums since the standard error of the estimates is large. Even so, there are cases where I reject the equality of the two price estimates. In some instances I reject the equality across groups where the risk premium was significant in one of the subsamples. Looking at Panel A2, I notice that the UI factor

was significantly priced for one of the subsamples when I sorted by size. Yet, when I compare it's prices across subsamples I reject the null of equality. When sorting by industry there is weak evidence of the same phenomena.

Looking across the various models and time periods, a few patterns emerge.[53] For example, the equality of URP's risk premium across samples is rejected in roughly one-half of the cases and I rarely reject the equality of DEI's risk premium. The evidence presented suggests that even when a factor is significant, it's price may not be the same across subsamples and therefore should not be considered a pervasive factor.

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[53]Only a few of the cases are presented in Table 6-14. I ran the tests for all models and time periods. The results are similar with the exception of the 1986-1990 time period (which are not reported).

CHAPTER 7

SUMMARY AND CONCLUSIONS

This dissertation develops and empirically tests two techniques for examining the robustness of factors in an APT context. The underlying logic is simple - if the APT holds all securities will be affected by the same factors and the prices of the factors will be the same for any security. Therefore if a large portfolio of stocks is divided into two groups the factors and prices estimated from the groups should be the same. Brown and Weinstein (1983) apply a similar logic in their test of the APT. My methodology, however, offers many advantages over previous tests.

Summarizing the results, I find that the asymptotic principal components technique is able to extract the true factors and the factor regression technique is able to compare the factors with little error. The results are sensitive to sample size, particularly in those models that allow each security to have a unique variance. The empirical evidence suggests that the real world data is not generated by any of the models I compare it to and there appears to be only one or two pervasive factors. The prices of risk for the macroeconomic variables are generally insignificant. In some cases a proxy is priced in one sample but the price is not the same across subsamples.

In order to compare the factors I first have to extract them from the time-series of returns data. Most of the previous work in this area has relied on factor analysis and a small number of securities in each portfolio. I rely on Connor and Korajczyk's (1986, 1988) asymptotic principal component technique which allows me

to use a large number of securities in my analysis. Given the asymptotic nature of the APT it is imperative to use a large sample.

I develop a new methodology, again based on the techniques of Connor and Korajczyk, for directly comparing the factors extracted from different portfolios. To my knowledge no one has ever used this methodology to compare factors extracted from different portfolios of securities. In fact, most of the previous studies have not actually compared the factors but rather some by-product or implication of the factors. For example, Brown and Weinstein (1983) compare the estimates of error variances obtained when the factors are extracted from subsets to those when they are extracted from the entire group. Although a common set of factors would pass a test like this, so would many other sets of factors that were totally different - as long as they explained a similar portion of the cross-sectional variance. My methodology compares the factors directly and this is a critical advantage.

Another advantage of the factor comparison technique is that it is a test of the primary assumption of the APT. As such, it is not subject to some of the criticisms associated with tests of the APT's pricing relationship. Since the APT is only an approximation, any empirical test involves additional assumptions in order to yield a strict equilibrium model. A common assumption is that there is some portfolio that is mean-variance efficient relative to the set of factors chosen. These assumptions complicate the tests because the tests then become a joint test of the APT and the additional assumptions.

For comparing the prices I use macroeconomic proxies for the factors. Numerous researchers have used proxies such as the ones I use for a variety of tests involving the APT. Several papers have compared risk premiums estimated across different time periods. No one has compared the prices from two samples over the same time period. This is a much cleaner test of the APT as one may reasonably expect the factors and their prices to vary over time.

For the factor comparison section I develop five models that generate security returns under varying assumptions. Two of the models are essentially those used by Connor and Korajczyk (1988, 1993). The other three are extensions of their work and offer their own contributions. Previous models have assumed either a strict factor structure or an economy in which the idiosyncratic return of every security is correlated (across securities but not intertemporally). I develop a model which generates the idiosyncratic return based on the sample's covariance matrix. If the returns are generated by an approximate factor structure, as the asymptotic principal components technique assumes, this is an important feature of my work.

Next, I demonstrate that the asymptotic principal components method and the factor regression method I use to compare factors are robust in their ability relative to the five models. I do this by comparing the extracted factors to the true factors for a variety of sample sizes. These results are interesting in their own right as they clearly demonstrate that comparing factors based on small portfolios is virtually meaningless. Another important result from these tests is that the rotational indeterminacy problem associated with the asymptotic principal components techniques does not affect my methodologies ability to identify them as pervasive factors.

Then, using the factor regression technique, I compare the R^2 values obtained from subsets of simulated data to those obtained from subsets of the real world data. The results indicate that the returns from the real world data are clearly not generated by a five factor model where the five factors are equally important and have equal idiosyncratic variances. This is the model used by Connor and Korajczyk (1993) to test their methodology for identifying the number of statistically priced factors. As such, one has to question if their technique is valid on real world data.

More importantly, I conclude that the returns data is characterized by at most two, and maybe only one, pervasive factor. Using a model that simulates returns

based on the five most dominant factors in the real world data, I find that the R^2 values obtained for the first and second factor are insignificantly different than those from the real data for the first and second factor. The third and fourth factor from the simulated returns have significantly higher R^2 values than the real world data.[54] Previous researchers have suggested that similar findings are due to the rotational indeterminacy of the techniques. My simulation results discussed above should partially alleviate these fears.

The results for the equality of prices across subsamples are not as strong as for the factor comparisons, but still offer some contributions. As documented by numerous other researchers, I find that the prices are very unstable and for the most part insignificant. The prices differ across time periods and across models. This variation is somewhat understandable. Changing economic conditions may change the factor structure and the correlation of the proxies will impact all the prices as various proxies are added to or removed from the model.

The difference across subsamples for the same time period and using the exact same model, which is a unique aspect of my research, is somewhat disturbing. There are two possible explanations and I think both of them contribute to my findings. First, returns data for securities, even after grouping them into portfolios, are very noisy. Second, there are thousands of different potential macroeconomic proxies and even after choosing the "correct" proxy, one has to specify the innovation in that variable and how to align the time-series with the returns data. If there are only one or two factors, as the previous sections suggest, the choice of identifying the right one is seemingly impossible!

Nonetheless, the methodology I presented offers another technique to screen potential proxies and it allows one to make inferences using the entire sample period.

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[54]The R^2 value for the fifth factor is also higher. Due to large standard errors for it and the R^2 value from the actual data's fifth factor the difference is insignificant.

In other words, if a factor is found to be significant over some time period this test is another way to identify if it is indeed a pervasive factor.

As the previous paragraphs indicate, the results of this dissertation already offer several significant contributions to the field of finance. However, there are several theoretical and empirical areas that warrant further investigation. These suggested recommendations are discussed in the following paragraphs.

It appears that, especially for the real world data, the R^2 values for the first and second factor are very polar - some are around 0.8 to 1.0 and others are around 0.0 to 0.2 and not very many fall in between. One explanation is that a security in the sample experienced a stream of large and unusual idiosyncratic shocks and that security returns dominate the true factor or factors. In the extreme a "Heywood" case occurs when one of the factors is near perfectly correlated with the returns of one security. Some researchers have thrown out these cases (an arguably dangerous approach). Another idea would be to increase the sample size to further "wash out" the idiosyncratic risk and reduce the chance of that security being sampled over the monte-carlo runs.

There might be some way to scale each asset's returns (both systematic and unsystematic) without affecting the factor structure. One idea, analogous to weighted least squares, is to divide each security return by the estimated standard deviation of it's error terms. By decreasing the idiosyncratic shocks, the true factor structure may be easier to identify.

The number of possible factor structures to explore is limitless and different models might offer more insight. Two suggested alternatives are: 1) to look at a one

or two factor model, and 2) to increase the number of securities in my fifth model which uses the full sample covariance matrix.[55]

For the pricing comparisons most of the enhancements would involve some advanced econometric technique. For example, weighted least squares, seemingly unrelated regressions, or Shanken's (1992) EIV correction could be applied. Even with these techniques many researchers have found few priced factors and price instability across groups. The use of macroeconomic proxies will always be popular because of their intuitive appeal. The proxies are valuable in helping explain what affected certain securities at certain times. I think the future payoff is in the work of those that are comparing the proxies to extracted factors.

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[55]This is a computationally time consuming task. It involves estimating a full $n \times n$ covariance matrix and then each random draw involves hundreds of thousands of calculations.

BIBLIOGRAPHY

Abeysekera, Sarath P., and Arvind Mahajan, 1987. "A Test of the APT in Pricing UK Stocks," *Journal of Business Finance and Accounting* 14, 377-391.

Abken, Peter A., Robert A. Jarrow, and Dilip B. Madan, 1996. "Testing the Ross APT on Options Data," Working Paper.

Admati, Anat R., and Paul Pfleiderer, 1985. "Interpreting the Factor Risk Premia in the Arbitrage Pricing Theory," *Journal of Economic Theory* 35, 191-195.

Ajayi, Richard A., and Mbodja Mougoue, 1996. "On the Dynamic Relation Between Stock Prices and Exchange Rates," *Journal of Financial Research* 19, 193-207.

Akaike, H., 1974. "A New Look at the Statistical Model Identification," *IEEE Transactions on Automatic Control* 19, 716-723.

Amihud, Yakov, Bent Jesper Christensen, and Haim Mendelson, 1992. "Further Evidence in the Risk-Return Relationship," Working Paper, New York University.

Bansal, Ravi, and S. Viswanathan, 1993. "No Arbitrage and Arbitrage Pricing: A New Approach," *Journal of Finance* 48, 1231-1262.

Bansal, Ravi, David A. Hsieh, and S. Viswanathan, 1993. "A New Approach to International Arbitrage Pricing," *Journal of Finance* 48, 1719-1747.

Banz, Rolf W., 1981. "The Relationship Between Return and Market Value of Common Stocks," *Journal of Financial Economics* 9, 3-18.

Basu, Sanjoy, 1983. "The Relationship Between Earnings' Yield, Market Value and Return for NYSE Common Stocks: Further Evidence," *Journal of Financial Economics* 12, 129-156.

Bartov, Eli, Gordon M. Bodnar, and Aditya Kaul, 1996. "Exchange Rate Variability and the Riskiness of U.S. Multinational Firms: Evidence from the Breakdown of the Bretton Woods System," *Journal of Financial Economics* 42, 105-132.

Beenstock, Michael, and Kam-Fai Chan, 1986. "Testing the Arbitrage Pricing Theory in the United Kingdom," *Oxford Bulletin of Economics and Statistics* 48, 121-141.

Beckers, Stan, Richard Grinold, Andrew Rudd, and D.Stefek, 1992. "The Relative Importance of Common Factors Across the European Equity Markets," *Journal of Banking and Finance* 16, 75-96.

Bhandari, Laxmi Chand, 1988. ""Debt/Equity Ratio and Expected Common Stock Returns: Empirical Evidence," *Journal of Finance* 43, 507-528.

Black, Fischer, 1993. "Beta and Return," *Journal of Portfolio Management* 20, 8-18.

Bower, Dorothy H., Richard S. Bower, and Dennis E. Logue, 1984. "Arbitrage Pricing Theory and Utility Stock Returns," *Journal of Finance* 39, 1041-1054.

Brown, Stephen J., 1988. "Comment," *Journal of Business and Economic Statistics* 6, 21-23.

Brown, Stephen J., 1989. "The Number of Factors in Security Returns," *Journal of Finance* 44, 1247-1262.

Brown, Stephen J., and Mark I. Weinstein, 1983. "A New Approach to Testing Asset Pricing Models: The Bilinear Paradigm," *Journal of Finance* 38, 711-743.

Brown, Stephen J., and Toshiyuki Otsuki, 1992. "Macroeconomic Factors and the Japanese Equity Markets: The CAPMD Project," in Edwin J. Elton and Martin J. Gruber, eds., *Japanese Capital Markets* (Harper and Row, New York, NY).

Booth, James R., and Lena Chua Booth, 1997. "Economic Factors, Monetary Policy, and Expected Returns on Stocks and Bonds," *Economic Review, Federal Reserve Bank of San Francisco* 2, 32-42.

Burmeister, Edwin, and Marjorie B. McElroy, 1988. "Joint Estimation of Factor Sensitivities and Risk Premia for the Arbitrage Pricing Theory," *Journal of Finance* 43, 721-733.

Burmeister, Edwin, and Kent D. Wall, 1986. "The Arbitrage Pricing Theory and Macroeconomic Factor Measures," *Financial Review* 21, 1-20.

Chamberlain, Gary, 1983. "Funds, Factors, and Diversification in Arbitrage Pricing Models," *Econometrica* 51, 1305-1323.

Chamberlain, Gary, and Michael Rothschild, 1983. "Arbitrage, Factor Structure, and Mean-Variance Analysis on Large Asset Markets," *Econometrica* 51, 1281-1304.

Chan, K.C., Nai-Fu Chen, and David A. Hsieh, 1985. "An Exploratory Investigation of the Firm Size Effect," *Journal of Financial Economics* 14, 451-471.

Chen, Nai-fu, 1983. "Some Empirical Test of the Arbitrage Pricing Theory," *Journal of Finance* 38, 1393-1414.

Chen, Nai-fu, 1988. "Comment," *Journal of Business and Economic Statistics* 6, 16.

Chen, Nai-fu, and Jonathon Ingersoll, 1983. "Exact Pricing in Linear factor Models with Finitely Many Assets," *Journal of Finance* 38, 985-988.

Chen, Nai-Fu, Richard Roll, and Stephen A. Ross, 1986. "Economic Forces and the Stock Market," *Journal of Business* 59, 383-403.

Cho, D. Chinhyung, 1984. "On Testing the Arbitrage Pricing Theory: Inter-Battery Factor Analysis," *Journal of Finance* 39, 1485-1502.

Cho, D. Chinhyung, and Simon J. Pak, 1991. "Multifactor Pricing Models with Macroeconomic Variables," in Son-nan Chen and Cheng-few Lee, eds., *Advances in Investments Analysis and Portfolio Management, Volume 1* (JAI Press, Greenwich, CN).

Cho, D. Chinyung, and William M. Taylor, 1987. "The Seasonal Stability of the Factor Structure of Stock Returns," *Journal of Finance* 42, 1195-1241.

Cho, D. Chinyung, Edwin J. Elton, and Martin J. Gruber, 1984. "On the Robustness of the Roll and Ross Arbitrage Pricing Theory," *Journal of Financial and Quantitative Analysis* 19, 1-10.

Chow, Edward H., Wayne Y. Lee, and Michael E. Soft, 1997. "The Exchange-Rate Risk Exposure of Asset Returns," *Journal of Business* 70, 105-123.

Clare, Andrew D., and Stephen H. Thomas, 1994. "Macroeconomic Factors, the APT and the UK Stockmarket," *Journal of Business Finance and Accounting* 21, 309-330.

Clare, Andrew, Richard Priestley, and Stephen Thomas, 1997. "The Robustness of the APT to Alternative Estimators," *Journal of Business Finance and Accounting* 24, 645-655.

Connor, Gregory, 1984. "A Unified Beta Pricing Theory," *Journal of Economic Theory* 34, 13-31.

Connor, Gregory, 1995. "The Three Types of Factor Models: A Comparison of Their Explanatory Power," *Financial Analysts Journal* 51, 42-46.

Connor, Gregory, and Robert A. Korajczyk, 1986. "Performance Measurement with the Arbitrage Pricing Theory," *Journal of Financial Economics* 15, 373-394.

Connor, Gregory, and Robert A. Korajczyk, 1988. "Risk and Return in Equilibrium APT: Application of a New Test Methodology," *Journal of Financial Economics* 21, 255-289.

Connor, Gregory, and Robert A. Korajczyk, 1993. "A Test for the Number of Factors in an Approximate Factor Model," *Journal of Finance* 48, 1263-1291.

Connor, Gregory, and Robert A. Korajczyk, 1995. "The Arbitrage Pricing Theory and Multifactor Models of Asset Returns," in R.A. Jarrow, V. Maksimovic, and W.T. Ziemba, eds. Handbooks in OR & MS, Vol. 9 (Elsevier Science B.V., Amsterdam).

Conover, Teresa L., 1997. "A Comparative Analysis of the Market Model and the Multiple-Factor Market Model," *Journal of Business Finance and Accounting* 24, 657-665.

Conway, Delores A., and Marc R. Reinganum, 1988a. "Stable Factors in Security Returns: Identification Using Cross-Validation," *Journal of Business and Economic Statistics* 6, 1-15.

Conway, Delores A., and Marc R. Reinganum, 1988b. "Reply," *Journal of Business and Economic Statistics* 6, 24-28.

Cragg, John G., and Burton G. Malkiel, 1982. "Expectations and the Structure of Share Prices," Working Paper, University of Chicago.

Daniel, Kent, and Sheridan Titman, 1997. "Evidence on the Characteristics of Cross Sectional Variation in Stock Returns," *Journal of Finance* 52, 1-33.

Dhrymes, Phoebus J., Irwin Friend, and N. Bulent Gultekin, 1984. "A Critical Reexamination of the Empirical Evidence on the Arbitrage Pricing Theory," *Journal of Finance* 39, 323-350.

Dhrymes, Phoebus J., Irwin Friend, Mustafa N. Gultekin, and N. Bulent Gultekin, 1985a. "New Tests of the APT and Their Implications," *Journal of Finance* 40, 659-675.

Dhrymes, Phoebus J., Irwin Friend, N. Bulent Gultekin, and Mustafa N. Gultekin, 1985b. “An Empirical Examination of the Implications of Arbitrage Pricing Theory,” *Journal of Banking and Finance* 9, 73-99.

Dybvig, Philip H., 1983. "An Explicit Bound on Individual Assets Deviations From APT Pricing in a Finite Economy," *Journal of Financial Economics* 12, 483-496.

Dybvig, Philip H., and Stephen A. Ross, 1985. “Yes, the APT is Testable,” *Journal of Finance* 40, 1173-1188.

Ehrhardt, Michael C., 1987. “Arbitrage Pricing Models: The Suffcient Number of Factors and Equilibrium Conditions,” *Journal of Financial Research* 10, 111-120.

Elton, Edwin J., Martin J. Gruber, and Christopher R. Blake, 1995. “Fundamental Economic Variables, Expected Returns, and Bond Fund Performance,” *Journal of Finance* 50, 1229-1256.

Engle, Robert F., Victor K. Ng, and Michael Rothschild, 1990. “Asset Pricing with a Factor-Arch Covariance Structure,” *Journal of Econometrics* 45, 213-237.

Epps, T.W., and Charles F. Kramer, 1996. “A Variance Test of the Linear Factor Representation of Returns and Equilibrium Pricing Restrictions,” Working Paper.

Fama, Eugene F., 1991. “Efficient Capital Markets: II,” *Journal of Finance* 46, 1575-1617.

Fama, Eugene F., 1996. “Multifactor Portfolio Efficiency and Multifactor Asset Pricing,” *Journal of Financial and Quantitative Analysis* 31, 441-465.

Fama, Eugene F., 1997. “Determining the Number of Priced State Variables in the ICAPM and the APT,” CRSP Working Paper 447.

Fama, Eugene F., and James D. MacBeth, 1973. "Risk, Return, and Equilibrium: Empirical Tests," *Journal of Political Economy* 81, 607-636.

Fama, Eugene F., and Kenneth R. French, 1992. “The Cross-Section of Expected Stock Returns,” *Journal of Finance* 47, 427-465.

Fama, Eugene F., and Kenneth R. French, 1993. “Common Risk Factors in the Returns on Stocks and Bonds,” *Journal of Financial Economics* 33, 3-56.

Fama, Eugene F., and Kenneth R. French, 1996a. "The CAPM is Wanted, Dead or Alive," *Journal of Finance* 51, 1947-1958..

Fama, Eugene F., and Kenneth R. French, 1996b. "Multifactor Explanations of Asset Pricing Anomalies," *Journal of Finance* 51, 55-84.

Fama, Eugene F., and Kenneth R. French, 1997. "Industry Costs of Equity," *Jouornal of Financial Economics* 43, 153-193.

Fama, Eugene F., and Michael R. Gibbons, 1984. "A Comparison of Inflation Forecasts," *Journal of Monetary Economics* 13, 327-348.

Ferson, Wayne E., and Campbell R. Harvey, 1991. "The Variation of Economic Risk Premiums," *Journal of Political Economy* 99, 385-415.

Ferson, Wayne E., and Ravi Jagannathan, 1996. "Econometric Evaluation of Asset Pricing Models," Staff Report 206, Federal Reserve Bank of Minneapolis.

Ferson, Wayne E., and Robert A. Korajczyk, 1995. "Do Arbitrage Pricing Models Explain the Predictability of Stock Returns?," *Journal of Business* 68, 309-349.

Flannery, Mark J., Allaudeen S. Hameed, and Richard H. Harjes, 1997. "Asset Pricing, Time-Varying Risk Premia and Interest Rate Risk," *Journal of Banking and Finance* 21, 315-335.

Fogler, H. Russell, Kose John, and James Tipton, 1981. "Three Factors, Interest Rate Differentials and Stock Groups," *Journal of Finance* 36, 323-335.

Gangopadhyay, Partha, 1996. "Macroeconomic Variables and Seasonal Mean Reversion in Stock Returns," *Journal of Financial Research* 19, 395-416.

Garrett, Ian, and Richard Priestley, 1997. "Do Assumptions About Factor Structure Matter in Empirical Tests of the APT?" *Journal of Business Finance and Accounting* 24, 249-260.

Ghysels, Eric, 1997. "On Stable Factor Structures in the Pricing of Risk: Do Time Varying Betas Help or Hurt," Working Paper.

Gibbons, Michael R., 1982. "Multivariate Tests of Financial Models," *Journal of Financial Economics* 10, 3-27.

Grinblatt, Mark, and Sheridan Titman, 1983. "Factor Pricing in a Finite Economy," *Journal of Financial Economics* 12, 497-507.

Grinblatt, Mark, and Sheridan Titman, 1985. "Approximate Factor Structures: Interpretations and Implications for Empirical Tests," *Journal of Finance* 85, 1367-1373.

Grinblatt, Mark, and Sheridan Titman, 1987. "The Relation Between Mean-Variance Efficiency and Arbitrage Pricing," *Journal of Business* 60, 97-112.

Gultekin, Mustafa N., and N. Bulent Gultekin, 1987. "Stock Return Anomalies and the Tests of the APT," *Journal of Finance* 42, 1213-1224.

Gweke, John, and Guofo Zhou, 1996. "Measuring the Pricing Error of the Arbitrage Pricing Theory," *Review of Financial Studies* 9, 557-587.

Hamao, Yasushi, 1988. "An Empirical Investigation of the Arbitrage Pricing Theory," *Japan and the World Economy* 1, 45-61.

Hamao, Yasushi, 1992. "An Empirical Investigation of the Arbitrage Pricing Theory: Using Japanese Data," in Edwin J. Elton and Martin J. Gruber, eds., *Japanese Capital Markets* (Harper and Row, New York, NY).

Handa, Puneet, 1991. "Arbitrage Pricing in a Finite Economy When There is Differential Parameter Uncertainty," Son-nan Chen and Cheng-few Lee, eds., *Advances in Investments Analysis and Portfolio Management, Volume 1* (JAI Press, Greenwich, CT).

Handa, Puneet, and Scott C. Linn, 1991. "Equilibrium Factor Pricing with Heterogeneous Beliefs," *Journal of Financial and Quantitative Analysis* 26, 11-22.

Handa, Puneet, and Scott C. Linn, 1993. "Arbitrage Pricing with Estimation Risk," *Journal of Financial and Quantitative Analysis* 28, 81-100.

Haugen, Robert A., and Nardin L. Baker, 1996. "Commonality in the Determinants of Expected Stock Returns," *Journal of Financial Economics* 41, 401-439.

He, Jia, and Lillian K. Ng, 1994. "Economic Forces, Fundamental Variables, and Equity Returns," *Journal of Business* 67, 599-609.

He, Jia, Raymond Kan, Lilian Ng, and Chu Zhang, 1996. "Tests of the Relations Among Marketwide Factors, Firm-Specific Variables, and Stock Returns Using a Conditional Asset Pricing Model," *Journal of Finance* 51, 1891-1908.

Huang, Roger D., and Hoje Jo, 1992. "Transformed Securities and Alternative Factor Structures," *Journal of Finance* 47, 397-405.

Huang, Roger D., and Hoje Jo, 1995. "Data Frequency and the Number of Factors in Stock Returns," *Journal of Banking and Finance* 19, 987-1003.

Huberman, Gur, 1982. "A Simple Approach to Arbitrage Pricing Theory," *Journal of Economic Theory* 28, 183-191.

Ingersoll, Jonathan E., 1984. "Some Results in the Theory of Arbitrage Pricing," *Journal of Finance* 39, 1021-1039.

Jagannathan, Ravi, and Ellen R. McGrattan, 1995. "The CAPM Debate," *Federal Reserve Bank of Minneapolis, Quarterly Review*, 2-17.

Jagannathan, Ravi, and Zhenyu Wang, 1996. "The Conditional CAPM and the Cross-Section of Expected Returns," *Journal of Finance* 51, 3-53.

Jarrow, Robert A., 1988. "Preferences, Continuity, and the Arbitrage Pricing Theory," *The Review of Financial Studies* 1, 159-172.

Jobson, J. D., 1982. "A Multivariate Linear Regression Test for the Arbitrage Pricing Theory," *Journal of Finance* 37, 1037-1042.

Jobson, J. D., 1988. "Comment," *Journal of Business and Economic Statistics* 6, 17-20.

Jorion, Philippe, 1991. "The Pricing of Exchange Rate Risk in the Stock Market," *Journal of Financial and Quantitative Analysis* 26, 363-376.

Keim, Donald B., 1983. "Size-Related Anomalies and Stock Return Seasonality: Further Empirical Evidence," *Journal of Financial Economics* 12, 13-32.

Kim, Dongcheol, 1995. "The Errors in Variables Problem in the Cross-Section of Expected Stock Returns," *Journal of Finance* 50, 1605-1634.

Kim, Moon K., and Chunchi Wu, 1987. "Macro-Economic Factors and Stock Returns," *Journal of Financial Research* 10, 87-98.

Knez, Peter J., and Mark J. Ready, 1997. "On the Robustness of Size and Book-To-Market in Cross-Sectional Regressions," Working Paper.

Kothari, S. P., Jay Shanken, and Richard G. Sloan, 1995. "Another Look at the Cross-Section of Expected Returns," *Journal of Finance* 50, 185-224.

Koutoulas, George, and Lawrence Kryzanowski, 1996. "Macrofactor Conditional Volatilities, Time-Varying Risk Premia and Stock Return Behavior," *Financial Review* 31, 169-195.

Kramer, Charles, 1994. "Macroeconomic Seasonality and the January Effect," *Journal of Finance* 49, 1883-1891.

Kryzanowski, Lawrence, and Minh Chau To, 1983. "General Factor Models and the Structure of Security Returns," *Journal of Financial and Quantitative Analysis* 18, 31-52.

Kryzanowski, Lawrence, Simon Lalancette, and Minh Chau To, 1997. "Performance Attribution using an APT with Prespecified Macrofactors and Time-Varying Risk Premia and Betas," *Journal of Financial and Quantitative Analysis* 32, 205-224.

Lee, Cheng F., Hun Y. Park, and K.C. John Wei, 1993. "An Equilibrium APT and the Role of the Market Portfolio," in Cheng F. Lee, ed., *Advances in Quantitative Analysis of Finance and Accounting, Volume 3, Part B* (JAI Press, Greenwich, CT).

Lehmann, Bruce N., and David M. Modest, 1988. "The Empirical Foundations of the Arbitrage Pricing Theory," *Journal of Financial Economics* 21, 213-254.

Lintner, John, 1965. "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *Review of Economics and Statistics* 47, 13-37.

Loughran, Tim, 1997. "Book-to-Market Across Firm Size, Exchange, and Seasonality: Is There an Effect," *Journal of Financial and Quantitative Analysis*, forthcoming.

McCulloch, Robert, and Peter E. Rossi, 1990. "Posterior, Predictive, and Utility-Based Approaches to Testing the Arbitrage Pricing Theory," *Journal of Financial Economics* 28, 7-38.

McCulloch, Robert, and Peter E. Rossi, 1991. "A Bayesian Approach to Testing the Arbitrage Pricing Theory," *Journal of Econometrics* 49, 141-168.

McGowan, Carl B., and Jack Clark Francis, 1991. "Arbitrage Pricing Theory Factors and their Relationship to Macroeconomic Variables," in Cheng F. Lee, Thomas J. Frecka, and Louis O. Scott, eds., *Advances in Quantitative Analysis of Finance and Accounting, Volume 1, Part A* (JAI Press, Greenwich, CT)

McElroy, Marjorie B., and Edwin Burmeister, 1988. "Arbitrage Pricing Theory as a Restricted Nonlinear Multivariate Regression Model," *Journal of Business and Economic Statistics* 6, 29-42.

Megginson, William L., 1997. *Corporate Finance Theory*. (Addison-Wesley, Reading, MA).

Mei, Jianping, 1993a. "A Semiautoregression Approach to the Arbitrage Pricing Theory," *Journal of Finance* 38, 599-620.

Mei, Jianping, 1993b. "Explaining the Cross-Section of Returns via a Multi-Factor APT Model," *Journal of Financial and Quantitative Analysis* 28, 331-345.

Mossin, Jan, 1966. "Equilibrium in a Capital Asset Market," *Econometrica* 24, 768-783.

Mukherjee, Tarun K., and Atsuyuki Naka, 1995. "Dynamic Relations Between Macroeconomic Variables and the Japanese Stock Market: An Application of a Vector Error Correction Model," *Journal of Financial Research* 18, 223-237.

Priestley, Richard, 1996. "The Arbitrage Pricing Theory, Macroeconomic and Financial Factors, and Expectations Generating Processes," *Journal of Banking and Finance* 20, 869-890.

Reinganum, Marc R., 1981. "The Arbitrage Pricing Theory: Some Simple Tests," *Journal of Finance* 36, 313-322.

Reisman, Haim, 1992. "Reference Variables, Factor Structure, and the Approximate Multibeta Representation," *Journal of Finance* 47, 1303-1314.

Roll, Richard, 1977. "A Critique of the Asset Pricing Theory's Tests; Part I: On Past and Potential Testability of the Theory," *Journal of Financial Economics* 4, 129-176.

Roll, Richard, and Stephen A. Ross, 1980. "An Empirical Investigation of the Arbitrage Pricing Theory," *Journal of Finance* 35, 1073-1103.

Roll, Richard, and Stephen A. Ross, 1984. "A Critical Reexamination of the Empirical Evidence on the Arbitrage Pricing Theory: A Reply," *Journal of Finance* 39, 347-350.

Rosenberg, Barr, 1974. "Extra-Market Components of Covariance in Security Returns," *Journal of Financial and Quantitative Analysis* 9, 263-273.

Ross, Stephen A., 1976. "Theory of Capital Asset Pricing," *Journal of Economic Theory,* 341-360.

Ross, Stephen A., 1977. "Risk, Return, and Arbitrage," in Irwin Friend and James L. Micksler, eds., *Risk and Return in Finance, Volume 1* (Ballinger, Cambridge, MA).

Ross, Stephen A., 1984. "Reply to Dhrymes: APT is Empirically Relevant," *Journal of Portfolio Management* 11, 54-56.

Schwartz, G., 1978. "Estimating the Dimension of a Model," *The Annuals of Statistics* 6, 461-464.

Shanken, Jay, 1982. "The Arbitrage Pricing Theory: Is it Testable," *Journal of Finance* 37, 1129-1140.

Shanken, Jay, 1985. "Multi-Beta CAPM or Equilibrium APT?: A Reply," *Journal of Finance* 40, 1189-1196.

Shanken, Jay, 1992a. "The Current State of the Arbitrage Pricing Theory," *Journal of Finance* 47, 1569-1574.

Shanken, Jay, 1992b. "On the Estimation of Beta Pricing Models," *Review of Finacial Studies* 5, 1-33.

Shanken, Jay, and Clifford W. Smith, 1996. "Implications of Capital Markets Research for Corporate Finance," *Financial Management* 25, 98-104.

Sharpe, William F., 1964. "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," *Journal of Finance* 19, 425-442.

Shukla, Ravi, and Charles Trzcinka, 1990. "Sequential Tests of the Arbitrage Pricing Theory: A Comparison of Principal Components and Maximum Likelihood Factors," *Journal of Finance* 45, 1541-1564.

Stambaugh, Robert F., 1988. "Comment," *Journal of Business and Economic Statistics* 6, 20-21.

Trzcinka, Charles, 1986. "On the Number of Factors in the Arbitrage Pricing Model," *Journal of Finance* 41, 347-368.

Velu, Raja, and Ghuofu Zhou, 1996. "Testing Multi-Beta Asset Pricing Models," Working Paper.

Warga, Arthur D., 1989. "Experimental Designs in Tests of Linear Factor Models," *Journal of Business and Economic Statistics* 7, 191-198.

Wei, K.C. John, 1988. "An Asset-Pricing Theory Unifying the CAPM and APT," *Journal of Finance* 43, 881-892.

Young, S. David, Michael A. Berry, David W. Harvey, and John R. Page, 1987. "Systematic Risk and Accounting Information Under the Arbitrage Pricing Theory," *Financial Analysts Journal* 43, 73-76.

Young, S. David, Michael A. Berry, David W. Harvey, and John R. Page, 1991. "Macroeconomic Forces, Systematic Risk, and Financial Variables: An Empirical Investigation," *Journal of Financial and Quantitative Analysis* 26, 559-564.

Zhou, Guofu, 1996. "Security Return Factors as Linear Combinations of Economic Variables," Working Paper.

APPENDIX A

TABLES

TABLE 5-1

Time-Series Parameters for Macroeconomic Proxies
Calculated from 15 years of monthly data (1981 - 1995)

This table reports the mean, standard deviation, and first 12 autocorrelation coefficients for the various proxies.

Proxy	mean	s.d.	r(1)	r(2)	r(3)	r(4)	r(5)	r(6)	r(7)	r(8)	r(9)	r(10)	r(11)	r(12)
MP	0.0019	0.0066	0.217	0.227	0.220	0.207	0.095	0.048	0.062	0.145	0.042	-0.030	0.074	-0.069
UI	-0.0002	0.0020	0.177	-0.760	-0.052	-0.133	-0.131	-0.065	-0.021	0.027	0.146	0.088	0.134	-0.035
DEI	-0.0001	0.0008	-0.077	-0.172	-0.007	-0.008	-0.810	-0.024	-0.014	-0.007	0.120	0.023	0.132	-0.031
URP	-0.0002	0.0113	-0.157	-0.540	0.017	0.042	-0.019	0.048	-0.119	0.138	-0.064	-0.075	0.102	-0.050
URP-2	-0.0037	0.0755	0.166	-0.233	-0.078	0.021	0.097	-0.120	-0.064	0.018	-0.102	0.082	0.102	-0.077
UTS	0.0054	0.0330	0.042	-0.017	-0.090	0.042	0.045	0.070	-0.049	-0.027	-0.017	0.044	0.119	-0.076
EW	0.0139	0.0487	0.355	0.012	-0.004	-0.039	0.027	0.029	-0.029	-0.138	-0.049	0.113	0.085	0.059

Note: s.d. is the standard deviation and r(i) is the correlation coefficient for the i^{th} lag.

TABLE 5-2

Correlation Coefficients for Macroeconomic Proxies
Calculated from 15 years of monthly data (1981 - 1995)

Variable	MP	UI	DEI	URP	URP-2	UTS	EW
MP	1.0000						
UI	0.1385	1.0000					
DEI	0.0302	0.2052	1.0000				
URP	-0.0001	0.2787	0.0048	1.0000			
URP-2	-0.2949	-0.2166	-0.2078	-0.2517	1.0000		
UTS	-0.1865	-0.0247	-0.1572	-0.5849	0.5482	1.0000	
EW	0.0107	-0.1892	-0.1052	-0.0249	0.0000	0.2153	1.0000

TABLE 6-1

Comparison of the Five Models Used to Generate Simulated Returns

#	Based On	Betas (B)	Factors (F)	Residuals	Comments
1	Connor & Korajczyk (1988) - they use monthly data.	Estimated via OLS regressions (so same for each iteration)	Extracted (so same for each iteration)	Diagonal covariance matrix plus correlation parameter. (strict or "pseudo-approximate" structure)	Rho sets correlation for all securities (cross-sectional)
2	Connor & Korajczyk (1993) - they use monthly data.	Simulated - i.i.d.	Simulated - i.i.d.	Simulated - i.i.d. (strict structure)	Uses market and sample parameters to set mean and variance for B & F
3	Model 2	Simulated - i.i.d.	Simulated - i.i.d.	Simulated - i.i.d. (strict structure)	Uses sample parameters to set mean and variance for B & F
4	Model 3	Simulated - independent but scaled (4x, 2x, 1x, 0.5x, 0.25x)	Simulated - i.i.d.	Simulated - i.i.d. (strict structure)	Uses sample parameters to set mean and variance for B & F
5	Model 3	Simulated - i.i.d.	Simulated - i.i.d.	Random draw from a multivariate normal based on full cross-sectional covariance matrix (approximate structure)	Limited sample size. Need n less than or equal to T. Use 10 year window and 500 securities.

TABLE 6-2

Comparing Factors Extracted From Two Mutually Exclusive Portfolios of Real Data

This table reports R^2 values obtained by regressing the factors extracted from sample number one on the seven factors extracted from sample two. The mean, maximum, and minimum R^2 value along with it's standard deviation (Stdev) across 100 iterations are reported. Each panel reports the results based on a different number of securities (n) in the portfolios.

	1986 - 1995				1986 - 1990				1991 - 1995			
Factor	Mean	Stdev	Min	Max	Mean	Stdev	Min	Max	Mean	Stdev	Max	Min
Panel A: n = 50												
1	0.253	0.198	0.003	0.656	0.298	0.235	0.007	0.697	0.061	0.054	0.001	0.277
2	0.195	0.167	0.005	0.613	0.246	0.215	0.003	0.748	0.080	0.073	0.013	0.407
3	0.107	0.132	0.005	0.676	0.103	0.102	0.004	0.512	0.083	0.078	0.009	0.378
4	0.061	0.083	0.004	0.428	0.066	0.067	0.007	0.455	0.080	0.072	0.005	0.309
5	0.040	0.044	0.001	0.261	0.047	0.038	0.005	0.239	0.065	0.051	0.006	0.254
6	0.029	0.028	0.002	0.162	0.046	0.035	0.005	0.269	0.053	0.042	0.009	0.290
7	0.023	0.017	0.003	0.135	0.040	0.035	0.006	0.209	0.054	0.048	0.004	0.278
Panel B: n = 150												
1	0.579	0.315	0.006	0.857	0.706	0.270	0.012	0.879	0.241	0.210	0.004	0.658
2	0.210	0.278	0.006	0.830	0.172	0.266	0.004	0.854	0.200	0.175	0.008	0.662
3	0.064	0.124	0.007	0.756	0.049	0.034	0.007	0.247	0.112	0.121	0.004	0.534
4	0.022	0.017	0.002	0.132	0.048	0.032	0.007	0.146	0.094	0.118	0.004	0.659
5	0.022	0.014	0.002	0.067	0.049	0.031	0.005	0.181	0.066	0.070	0.010	0.409
6	0.017	0.010	0.002	0.048	0.040	0.026	0.006	0.113	0.043	0.037	0.008	0.229
7	0.018	0.014	0.003	0.082	0.040	0.021	0.006	0.112	0.049	0.033	0.008	0.158

Factor	1986 - 1995 Mean	Stdev	Min	Max	1986 - 1990 Mean	Stdev	Min	Max	1991 - 1995 Mean	Stdev	Max	Min
Panel C: $n = 500$												
1	0.725	0.393	0.004	0.955	0.929	0.045	0.792	0.963	0.496	0.356	0.004	0.883
2	0.231	0.388	0.003	0.948	0.114	0.104	0.004	0.739	0.281	0.304	0.007	0.885
3	0.023	0.022	0.003	0.122	0.105	0.111	0.007	0.688	0.109	0.193	0.007	0.859
4	0.034	0.030	0.004	0.192	0.078	0.066	0.006	0.354	0.057	0.103	0.004	0.748
5	0.041	0.040	0.003	0.208	0.089	0.057	0.013	0.277	0.040	0.032	0.007	0.170
6	0.031	0.024	0.003	0.098	0.076	0.059	0.007	0.316	0.038	0.025	0.001	0.137
7	0.032	0.030	0.005	0.192	0.068	0.056	0.004	0.380	0.036	0.022	0.003	0.105
Panel D: $n = 1500$ (1000)*												
1	0.557	0.469	0.012	0.976	0.983	0.002	0.978	0.987	0.640	0.432	0.006	0.960
2	0.438	0.465	0.001	0.964	0.261	0.269	0.009	0.773	0.322	0.433	0.002	0.953
3	0.044	0.056	0.003	0.283	0.338	0.233	0.012	0.750	0.020	0.012	0.002	0.060
4	0.091	0.092	0.005	0.410	0.283	0.186	0.023	0.711	0.023	0.016	0.002	0.082
5	0.097	0.088	0.008	0.421	0.254	0.149	0.011	0.627	0.031	0.024	0.003	0.137
6	0.091	0.103	0.004	0.484	0.171	0.109	0.009	0.509	0.040	0.032	0.004	0.195
7	0.070	0.065	0.003	0.236	0.152	0.092	0.010	0.448	0.038	0.031	0.004	0.137

* Note: For Panel D the full sample (1986-1995) $n = 1000$ due to the available sample size. For the two sub-periods $n = 1500$.

TABLE 6-3

Comparing Extracted Factors to True Factors Using the Entire Sample

This table reports R^2 values obtained by regressing the estimated factors (extracted from the simulated data) on the five true factors. The mean, maximum, and minimum R^2 value along with it's standard deviation (Stdev) across 25 iterations are reported. Each panel reports the results from a different simulation model (as described in Table 6.1). The number of individual simulated returns is equal to the number of securities in the full sample for each period. An R^2 value near one implies little error in the estimate.

		1986 - 1995				1986 - 1990				1991 - 1995			
rho	Factor	Mean	Stdev	Min	Max	Mean	Stdev	Min	Max	Mean	Stdev	Min	Max
Panel A: Model #1 - extracted factors, betas estimated from OLS regression, residual terms are random draws (diagonal covariance matrix plus correlation parameter - rho)													
0.00	1	0.994	0.000	0.993	0.994	0.995	0.001	0.994	0.996	0.996	0.000	0.995	0.997
	2	0.997	0.000	0.997	0.997	0.991	0.001	0.989	0.992	0.956	0.003	0.949	0.963
	3	0.827	0.033	0.758	0.869	0.851	0.031	0.783	0.918	0.930	0.015	0.906	0.953
	4	0.889	0.019	0.856	0.934	0.867	0.034	0.795	0.918	0.941	0.007	0.924	0.952
	5	0.832	0.039	0.765	0.911	0.836	0.036	0.733	0.905	0.892	0.009	0.870	0.907
	6	0.045	0.020	0.011	0.084	0.024	0.017	0.001	0.083	0.036	0.014	0.008	0.059
	7	0.042	0.022	0.006	0.082	0.012	0.010	0.000	0.043	0.005	0.006	0.000	0.021
0.25	1	0.968	0.003	0.962	0.972	0.993	0.001	0.992	0.995	0.995	0.001	0.994	0.995
	2	0.986	0.001	0.983	0.987	0.988	0.001	0.986	0.990	0.956	0.004	0.946	0.962
	3	0.815	0.042	0.719	0.866	0.857	0.026	0.795	0.900	0.934	0.011	0.904	0.954
	4	0.839	0.049	0.688	0.894	0.869	0.029	0.792	0.922	0.937	0.023	0.843	0.954
	5	0.770	0.035	0.708	0.846	0.830	0.038	0.758	0.913	0.896	0.016	0.871	0.932
	6	0.053	0.037	0.012	0.164	0.025	0.017	0.000	0.078	0.033	0.021	0.008	0.105
	7	0.051	0.034	0.008	0.111	0.013	0.012	0.000	0.049	0.003	0.003	0.000	0.009

		1986 - 1995				1986 - 1990				1991 - 1995			
rho	Factor	Mean	Stdev	Min	Max	Mean	Stdev	Min	Max	Mean	Stdev	Min	Max
Panel A: (continued)													
0.50	1	0.985	0.001	0.983	0.987	0.989	0.001	0.988	0.991	0.991	0.001	0.990	0.992
	2	0.989	0.001	0.988	0.991	0.984	0.002	0.980	0.986	0.956	0.004	0.948	0.962
	3	0.857	0.024	0.817	0.897	0.861	0.022	0.815	0.903	0.930	0.014	0.900	0.946
	4	0.877	0.035	0.778	0.927	0.869	0.030	0.801	0.920	0.941	0.008	0.924	0.953
	5	0.810	0.031	0.739	0.879	0.830	0.050	0.750	0.898	0.889	0.016	0.866	0.920
	6	0.057	0.021	0.022	0.104	0.035	0.027	0.003	0.108	0.040	0.016	0.022	0.077
	7	0.042	0.021	0.013	0.096	0.011	0.010	0.004	0.031	0.004	0.003	0.001	0.011
0.75	1	0.976	0.003	0.973	0.982	0.989	0.001	0.988	0.991	0.982	0.002	0.978	0.984
	2	0.973	0.010	0.945	0.985	0.984	0.002	0.980	0.986	0.945	0.016	0.897	0.963
	3	0.857	0.023	0.814	0.894	0.861	0.022	0.815	0.903	0.906	0.022	0.859	0.941
	4	0.861	0.031	0.803	0.907	0.869	0.030	0.801	0.920	0.921	0.015	0.874	0.937
	5	0.829	0.039	0.729	0.882	0.830	0.050	0.750	0.898	0.862	0.026	0.812	0.917
	6	0.098	0.036	0.032	0.158	0.035	0.027	0.003	0.108	0.080	0.041	0.031	0.169
	7	0.067	0.025	0.014	0.098	0.011	0.010	0.004	0.031	0.010	0.012	0.001	0.052
0.90	1	0.917	0.006	0.908	0.928	0.947	0.014	0.920	0.976	0.952	0.004	0.948	0.962
	2	0.983	0.002	0.978	0.986	0.948	0.015	0.910	0.980	0.812	0.087	0.622	0.901
	3	0.341	0.138	0.136	0.599	0.564	0.142	0.129	0.798	0.685	0.135	0.378	0.860
	4	0.245	0.135	0.085	0.634	0.595	0.156	0.210	0.801	0.691	0.170	0.246	0.854
	5	0.320	0.151	0.084	0.577	0.513	0.175	0.084	0.775	0.586	0.218	0.172	0.863
	6	0.267	0.143	0.070	0.549	0.296	0.170	0.052	0.726	0.575	0.173	0.178	0.822
	7	0.136	0.101	0.021	0.313	0.131	0.118	0.005	0.598	0.040	0.036	0.005	0.179

Factor	1986 - 1995				1986 - 1990				1991 - 1995			
	Mean	Stdev	Min	Max	Mean	Stdev	Min	Max	Mean	Stdev	Max	Min
Panel B: Model #2 - Factors, betas, and residuals are random draws from i.i.d. multivariate normal distributions. Parameters are tied to market and sample observations.												
1	0.995	0.000	0.995	0.996	0.997	0.000	0.996	0.998	0.995	0.001	0.995	0.997
2	0.981	0.002	0.978	0.985	0.985	0.001	0.981	0.987	0.987	0.001	0.985	0.989
3	0.979	0.002	0.975	0.982	0.982	0.002	0.979	0.987	0.985	0.002	0.980	0.988
4	0.977	0.002	0.974	0.980	0.980	0.002	0.977	0.984	0.983	0.002	0.979	0.986
5	0.975	0.003	0.968	0.979	0.977	0.002	0.974	0.981	0.980	0.003	0.970	0.985
6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.001
Panel C: Model #3 - Factors, betas, and residuals are random draws from i.i.d. multivariate normal distributions. Parameters are tied to sample observations.												
1	0.995	0.001	0.994	0.997	0.997	0.000	0.996	0.998	0.995	0.000	0.993	0.996
2	0.980	0.002	0.977	0.982	0.982	0.003	0.977	0.987	0.987	0.001	0.985	0.990
3	0.978	0.002	0.974	0.983	0.980	0.002	0.975	0.983	0.985	0.001	0.984	0.988
4	0.976	0.002	0.972	0.979	0.977	0.003	0.969	0.982	0.984	0.002	0.980	0.986
5	0.973	0.002	0.969	0.977	0.974	0.003	0.968	0.978	0.980	0.002	0.975	0.984
6	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.001
7	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.001

	1986 - 1995				1986 - 1990				1991 - 1995			
Factor	Mean	Stdev	Min	Max	Mean	Stdev	Min	Max	Mean	Stdev	Max	Min
Panel D: Model #4 - Factors, betas, and residuals are random draws from i.i.d. multivariate normal distributions. Parameters are tied to sample observations. Betas are scaled (4x, 2x, 1x, 0.5x, 0.25x).												
1	0.999	0.000	0.999	0.999	1.000	0.000	0.999	1.000	0.999	0.000	0.999	1.000
2	0.996	0.000	0.995	0.997	0.997	0.000	0.996	0.997	0.997	0.000	0.996	0.998
3	0.982	0.001	0.980	0.985	0.984	0.003	0.977	0.988	0.988	0.001	0.986	0.990
4	0.911	0.007	0.903	0.926	0.900	0.015	0.869	0.929	0.932	0.010	0.905	0.946
5	0.465	0.077	0.290	0.576	0.243	0.152	0.000	0.500	0.437	0.138	0.045	0.601
6	0.007	0.000	0.000	0.026	0.036	0.043	0.001	0.162	0.035	0.084	0.000	0.383
7	0.007	0.000	0.000	0.042	0.028	0.027	0.000	0.092	0.020	0.026	0.000	0.081
Panel E: Model #5 - Factors and betas are random draws from i.i.d. multivariate normal distributions. Residuals are drawn from multivariate normal based on covariance matrix (so approximate factor structure). Parameters are tied to sample observations.												
1	0.974	0.005	0.965	0.982	N/A				N/A			
2	0.368	0.375	0.004	0.914								
3	0.456	0.324	0.001	0.884								
4	0.453	0.277	0.012	0.877								
5	0.418	0.269	0.018	0.860								
6	0.281	0.217	0.004	0.675								
7	0.200	0.171	0.134	0.692								

Note: Panel E does not have results for the two five year intervals because Model #5 requires that the number of assets be less than the number of observations in the time-series of returns. Therefore, I only look at the ten year period so that sample size can be reasonably large.

TABLE 6-4

Comparing Extracted Factors to True Factors Using Small Portfolio (n = 50)

This table reports R^2 values obtained by regressing the factors extracted from simulated data on the five true factors. There are 50 securities in the simulated returns. The mean, maximum, and minimum R^2 value along with it's standard deviation (Stdev) across 25 iterations are reported. Each panel reports the results from a different simulation model.

	1986 - 1995				1986 - 1990				1991 - 1995			
Factor	Mean	Stdev	Min	Max	Mean	Stdev	Min	Max	Mean	Stdev	Max	Min
Panel A: Model #1 - extracted factors, betas estimated from OLS regression, residual terms are random draws (diagonal covariance matrix plus correlation parameter rho (rho = 0.5).												
1	0.607	0.230	0.052	0.991	0.614	0.225	0.019	0.953	0.236	0.190	0.011	0.993
2	0.183	0.222	0.003	0.873	0.266	0.214	0.030	0.799	0.174	0.152	0.020	0.618
3	0.079	0.099	0.006	0.618	0.123	0.099	0.008	0.529	0.160	0.125	0.019	0.625
4	0.063	0.078	0.002	0.483	0.106	0.073	0.012	0.320	0.123	0.098	0.009	0.548
5	0.050	0.052	0.003	0.333	0.091	0.060	0.010	0.312	0.095	0.066	0.010	0.339
6	0.049	0.038	0.003	0.238	0.073	0.054	0.003	0.297	0.080	0.058	0.011	0.294
7	0.037	0.029	0.003	0.159	0.056	0.037	0.003	0.244	0.060	0.046	0.004	0.258
Panel B: Model #3 - Factors, betas, and residuals are random draws from i.i.d. multivariate normal distributions. Parameters are tied to sample observations.												
1	0.826	0.022	0.772	0.865	0.832	0.029	0.750	0.891	0.749	0.039	0.654	0.830
2	0.559	0.052	0.387	0.657	0.473	0.068	0.239	0.597	0.538	0.067	0.269	0.657
3	0.473	0.052	0.370	0.607	0.370	0.084	0.109	0.543	0.449	0.066	0.251	0.600
4	0.388	0.068	0.199	0.505	0.265	0.093	0.030	0.468	0.346	0.094	0.085	0.492
5	0.268	0.101	0.008	0.462	0.172	0.089	0.015	0.395	0.207	0.110	0.006	0.419
6	0.029	0.030	0.001	0.182	0.081	0.067	0.003	0.299	0.068	0.061	0.004	0.259
7	0.027	0.031	0.003	0.231	0.051	0.039	0.005	0.176	0.046	0.034	0.004	0.152

	1986 - 1995				1986 - 1990				1991 - 1995			
Factor	Mean	Stdev	Min	Max	Mean	Stdev	Min	Max	Mean	Stdev	Max	Min
Panel C: Model #4 - Factors, betas, and residuals are random draws from i.i.d. multivariate normal distributions. Parameters are tied to sample observations. Betas are scaled (4x, 2x, 1x, 0.5x, 0.25x).												
1	0.968	0.006	0.949	0.979	0.966	0.007	0.945	0.979	0.957	0.011	0.905	0.972
2	0.840	0.030	0.753	0.898	0.814	0.039	0.691	0.891	0.831	0.037	0.723	0.910
3	0.505	0.078	0.231	0.684	0.438	0.101	0.041	0.593	0.467	0.104	0.108	0.700
4	0.072	0.064	0.002	0.267	0.049	0.041	0.003	0.241	0.060	0.051	0.001	0.252
5	0.033	0.032	0.001	0.164	0.031	0.026	0.001	0.115	0.036	0.030	0.002	0.125
6	0.025	0.021	0.001	0.118	0.039	0.035	0.000	0.238	0.032	0.029	0.001	0.156
7	0.017	0.015	0.001	0.073	0.028	0.022	0.001	0.121	0.029	0.024	0.001	0.105
Panel D: Model #5 - Factors and betas are random draws from i.i.d. multivariate normal distributions. Residuals are drawn from multivariate normal based on covariance matrix (so approximate factor structure). Parameters are tied to sample observations.												
1	0.527	0.336	0.004	0.924	N/A				N/A			
2	0.325	0.254	0.006	0.890								
3	0.254	0.224	0.006	0.870								
4	0.247	0.203	0.005	0.824								
5	0.226	0.191	0.009	0.768								
6	0.176	0.146	0.006	0.616								
7	0.159	0.133	0.003	0.551								

Note: Panel D does not have results for the two five year intervals because Model #5 requires that the number of assets be less than the number of observations in the time-series of returns.

TABLE 6-5

Comparing Extracted Factors to True Factors Using Small Portfolio (n = 150)

This table reports R^2 values obtained by regressing the factors extracted from simulated data on the five true factors. There are 150 securities in the simulated returns. The mean, maximum, and minimum R^2 value along with it's standard deviation (Stdev) across 25 iterations are reported. Each panel reports the results from a different simulation model.

	1986 - 1995				1986 - 1990				1991 - 1995			
Factor	Mean	Stdev	Min	Max	Mean	Stdev	Min	Max	Mean	Stdev	Max	Min
Panel A: Model #1 - extracted factors, betas estimated from OLS regression, residual terms are random draws (diagonal covariance matrix plus correlation parameter rho (rho = 0.5).												
1	0.825	0.200	0.174	0.991	0.904	0.062	0.315	0.958	0.574	0.261	0.032	0.994
2	0.299	0.343	0.004	0.958	0.295	0.265	0.020	0.928	0.350	0.274	0.008	0.831
3	0.101	0.149	0.002	0.863	0.159	0.112	0.006	0.532	0.160	0.152	0.009	0.757
4	0.055	0.042	0.004	0.215	0.147	0.111	0.006	0.633	0.106	0.073	0.007	0.427
5	0.049	0.040	0.004	0.181	0.132	0.094	0.015	0.435	0.081	0.059	0.005	0.284
6	0.053	0.051	0.003	0.261	0.110	0.072	0.002	0.315	0.082	0.051	0.012	0.294
7	0.050	0.044	0.005	0.253	0.091	0.070	0.004	0.354	0.076	0.061	0.004	0.366
Panel B: Model #3 - Factors, betas, and residuals are random draws from i.i.d. multivariate normal distributions. Parameters are tied to sample observations.												
1	0.933	0.007	0.912	0.945	0.938	0.008	0.913	0.954	0.892	0.017	0.836	0.927
2	0.771	0.024	0.706	0.823	0.725	0.033	0.623	0.788	0.769	0.028	0.708	0.821
3	0.737	0.025	0.671	0.786	0.673	0.043	0.549	0.765	0.723	0.033	0.645	0.792
4	0.700	0.030	0.608	0.768	0.613	0.055	0.465	0.724	0.675	0.041	0.557	0.757
5	0.655	0.037	0.554	0.747	0.547	0.060	0.346	0.707	0.613	0.051	0.479	0.729
6	0.005	0.004	0.001	0.021	0.018	0.014	0.002	0.060	0.012	0.008	0.001	0.034
7	0.004	0.003	0.000	0.019	0.015	0.011	0.001	0.054	0.011	0.008	0.001	0.051

Factor	1986 - 1995 Mean	Stdev	Min	Max	1986 - 1990 Mean	Stdev	Min	Max	1991 - 1995 Mean	Stdev	Max	Min
Panel C: Model #4 - Factors, betas, and residuals are random draws from i.i.d. multivariate normal distributions. Parameters are tied to sample observations. Betas are scaled (4x, 2x, 1x, 0.5x, 0.25x).												
1	0.989	0.001	0.985	0.992	0.989	0.002	0.984	0.992	0.985	0.003	0.974	0.990
2	0.943	0.007	0.923	0.962	0.932	0.011	0.905	0.955	0.936	0.011	0.910	0.963
3	0.776	0.028	0.663	0.835	0.734	0.046	0.586	0.826	0.748	0.038	0.629	0.849
4	0.289	0.110	0.005	0.515	0.114	0.091	0.001	0.355	0.171	0.098	0.004	0.401
5	0.031	0.045	0.001	0.283	0.050	0.053	0.000	0.321	0.047	0.044	0.001	0.232
6	0.022	0.024	0.000	0.126	0.046	0.042	0.001	0.210	0.033	0.033	0.001	0.179
7	0.018	0.017	0.001	0.106	0.030	0.028	0.001	0.151	0.031	0.029	0.001	0.130
Panel D: Model #5 - Factors and betas are random draws from i.i.d. multivariate normal distributions. Residuals are drawn from multivariate normal based on covariance matrix (so approximate factor structure). Parameters are tied to sample observations.												
1	0.860	0.191	0.032	0.967	N/A				N/A			
2	0.340	0.321	0.002	0.911								
3	0.359	0.286	0.003	0.896								
4	0.337	0.265	0.008	0.901								
5	0.313	0.232	0.006	0.853								
6	0.236	0.185	0.003	0.774								
7	0.185	0.151	0.003	0.544								

Note: Panel D does not have results for the two five year intervals because Model #5 requires that the number of assets be less than the number of observations in the time-series of returns. Therefore, I only look at the ten year period so that the sample size can be reasonably large.

TABLE 6-6

Comparing Extracted Factors to True Factors Using Small Portfolio (n = 500)

This table reports R^2 values obtained by regressing the factors extracted from simulated data on the five true factors. There are 500 securities in the simulated returns. The mean, maximum, and minimum R^2 value along with it's standard deviation (Stdev) across 25 iterations are reported. Each panel reports the results from a different simulation model.

	1986 - 1995				1986 - 1990				1991 - 1995			
Factor	Mean	Stdev	Min	Max	Mean	Stdev	Min	Max	Mean	Stdev	Max	Min
Panel A: Model #1 - extracted factors, betas estimated from OLS regression, residual terms are random draws (diagonal covariance matrix plus correlation parameter rho (rho = 0.5).												
1	0.968	0.012	0.949	0.992	0.970	0.005	0.940	0.978	0.894	0.098	0.364	0.994
2	0.634	0.361	0.030	0.968	0.681	0.203	0.215	0.959	0.530	0.301	0.013	0.936
3	0.311	0.260	0.018	0.952	0.500	0.194	0.077	0.947	0.253	0.210	0.011	0.907
4	0.135	0.124	0.007	0.549	0.372	0.192	0.028	0.728	0.167	0.139	0.002	0.854
5	0.128	0.136	0.008	0.670	0.197	0.143	0.006	0.717	0.130	0.098	0.008	0.369
6	0.079	0.097	0.002	0.585	0.143	0.115	0.002	0.558	0.107	0.075	0.009	0.346
7	0.051	0.052	0.001	0.285	0.110	0.092	0.005	0.409	0.093	0.070	0.009	0.370
Panel B: Model #3 - Factors, betas, and residuals are random draws from i.i.d. multivariate normal distributions. Parameters are tied to sample observations.												
1	0.979	0.002	0.973	0.984	0.981	0.003	0.970	0.987	0.966	0.004	0.953	0.976
2	0.916	0.007	0.896	0.933	0.898	0.012	0.869	0.921	0.917	0.011	0.877	0.937
3	0.906	0.008	0.887	0.924	0.882	0.012	0.846	0.905	0.902	0.012	0.852	0.930
4	0.896	0.009	0.869	0.916	0.865	0.015	0.816	0.901	0.886	0.014	0.847	0.920
5	0.881	0.010	0.852	0.910	0.837	0.024	0.738	0.882	0.865	0.015	0.822	0.898
6	0.001	0.001	0.000	0.005	0.003	0.003	0.000	0.014	0.003	0.002	0.000	0.008
7	0.001	0.001	0.000	0.004	0.003	0.003	0.000	0.014	0.002	0.001	0.000	0.007

Factor	1986-1995				1986-1990				1991 - 1995			
	Mean	Stdev	Min	Max	Mean	Stdev	Min	Max	Mean	Stdev	Max	Min
Panel C: Model #4 - Factors, betas, and residuals are random draws from i.i.d. multivariate normal distributions. Parameters are tied to sample observations. Betas are scaled (4x, 2x, 1x, 0.5x, 0.25x).												
1	0.997	0.000	0.995	0.998	0.997	0.001	0.995	0.998	0.996	0.001	0.994	0.997
2	0.982	0.002	0.976	0.986	0.979	0.003	0.969	0.986	0.980	0.003	0.971	0.986
3	0.921	0.008	0.897	0.939	0.903	0.016	0.850	0.938	0.914	0.013	0.875	0.951
4	0.674	0.038	0.572	0.750	0.506	0.111	0.116	0.681	0.582	0.072	0.424	0.717
5	0.028	0.026	0.000	0.120	0.035	0.050	0.000	0.332	0.029	0.031	0.001	0.176
6	0.022	0.027	0.000	0.142	0.026	0.024	0.000	0.127	0.029	0.026	0.000	0.120
7	0.021	0.025	0.000	0.127	0.025	0.022	0.000	0.109	0.021	0.024	0.000	0.121

Note: There is no Panel D (with a sample size of 500 the results are the same as in Table 6-3 which uses the full sample size).

TABLE 6-7

Comparing Extracted Factors to True Factors Using Small Portfolio (*n* = 1500)

This table reports R^2 values obtained by regressing the factors extracted from simulated data on the five true factors. There are 1500 securities in the simulated returns. The mean, maximum, and minimum R^2 value along with it's standard deviation (Stdev) across 25 iterations are reported. Each panel reports the results from a different simulation model.

	1986 - 1995				1986 - 1990				1991 - 1995			
Factor	Mean	Stdev	Min	Max	Mean	Stdev	Min	Max	Mean	Stdev	Max	Min
Panel A: Model #1 - extracted factors, betas estimated from OLS regression, residual terms are random draws (diagonal covariance matrix plus correlation parameter rho (rho = 0.5).												
1	0.982	0.007	0.972	0.991	0.987	0.001	0.983	0.990	0.976	0.013	0.958	0.994
2	0.828	0.213	0.082	0.980	0.909	0.070	0.627	0.960	0.843	0.144	0.424	0.972
3	0.573	0.299	0.005	0.951	0.845	0.084	0.642	0.951	0.631	0.214	0.039	0.932
4	0.321	0.265	0.012	0.841	0.805	0.079	0.430	0.912	0.415	0.226	0.005	0.764
5	0.131	0.159	0.005	0.741	0.463	0.335	0.003	0.886	0.200	0.164	0.007	0.705
6	0.082	0.088	0.001	0.496	0.089	0.110	0.002	0.643	0.154	0.114	0.008	0.495
7	0.042	0.040	0.001	0.257	0.034	0.039	0.001	0.185	0.104	0.087	0.003	0.365
Panel B: Model #3 - Factors, betas, and residuals are random draws from i.i.d. multivariate normal distributions. Parameters are tied to sample observations.												
1	0.989	0.001	0.986	0.992	0.994	0.001	0.990	0.995	0.988	0.002	0.983	0.992
2	0.955	0.004	0.945	0.964	0.963	0.004	0.954	0.973	0.969	0.003	0.961	0.977
3	0.950	0.004	0.939	0.961	0.957	0.005	0.944	0.968	0.965	0.004	0.955	0.973
4	0.946	0.005	0.934	0.960	0.951	0.005	0.933	0.961	0.959	0.005	0.947	0.971
5	0.939	0.006	0.923	0.952	0.943	0.007	0.926	0.957	0.953	0.007	0.935	0.965
6	0.000	0.000	0.000	0.002	0.001	0.001	0.000	0.004	0.001	0.001	0.000	0.003
7	0.001	0.000	0.000	0.003	0.001	0.001	0.000	0.004	0.001	0.000	0.000	0.002

	1986 - 1995				1986 - 1990				1991 - 1995			
Factor	Mean	Stdev	Min	Max	Mean	Stdev	Min	Max	Mean	Stdev	Max	Min

Panel C: Model #4 - Factors, betas, and residuals are random draws from i.i.d. multivariate normal distributions. Parameters are tied to sample observations. Betas are scaled (4x, 2x, 1x, 0.5x, 0.25x).

1	0.998	0.000	0.998	0.999	0.999	0.000	0.999	0.999	0.999	0.000	0.998	0.999
2	0.991	0.001	0.988	0.993	0.993	0.001	0.989	0.995	0.993	0.001	0.991	0.995
3	0.960	0.004	0.945	0.971	0.966	0.005	0.953	0.977	0.970	0.004	0.957	0.978
4	0.815	0.018	0.769	0.863	0.805	0.029	0.713	0.876	0.836	0.023	0.785	0.881
5	0.116	0.094	0.000	0.350	0.054	0.056	0.000	0.275	0.085	0.082	0.001	0.278
6	0.037	0.049	0.000	0.233	0.033	0.042	0.000	0.202	0.047	0.056	0.000	0.248
7	0.024	0.033	0.000	0.169	0.031	0.035	0.000	0.141	0.028	0.031	0.000	0.161

Note: There is no Panel D because the sample size is to large.

TABLE 6-8

Comparing the R^2 Values From Real Data and Simulated Data - (n = 50)

This table compares the R^2 values obtained from the sample data to the R^2 values obtained from simulated data. For each model I use a standard t-test and a Wilcoxon rank sum test to see if the R^2 values from the real world data are significantly different than the R^2 values obtained under the assumed models. I compare the first five factors. The Wilcoxon test is performed because most of the sample R^2 values do not appear to be normally distributed (as indicated by a Shapiro-Wilk W-test). The real world data in panel A is the same as in Table 6.2 the R^2 values in the other panels are the mean and standard deviation across 100 iterations of regressing the factors extracted from one simulated portfolio of returns on the factors extracted from another simulated portfolio of returns.

	1986 - 1995			1986 - 1990			1991 - 1995		
Factor	Mean	Stdev	p-value[1]	Mean	Stdev	p-value[1]	Mean	Stdev	p-value[1]
Panel A: Real World Data.									
1	0.253	0.198	<0.001	0.298	0.235	<0.001	0.061	0.054	<0.001
2	0.195	0.167	<0.001	0.246	0.215	<0.001	0.080	0.073	<0.001
3	0.107	0.132	<0.001	0.103	0.102	<0.001	0.083	0.078	<0.001
4	0.061	0.083	<0.001	0.066	0.067	<0.001	0.080	0.072	<0.001
5	0.040	0.044	<0.001	0.047	0.038	<0.001	0.065	0.051	<0.001

	1986 - 1995				1986 - 1990				1991 - 1995			
Factor	Mean	Stdev	t-stat[2]	z-stat[3]	Mean	Stdev	t-stat[2]	z-stat[3]	Mean	Stdev	t-stat[2]	z-stat[3]
Panel B: Model #1 - extracted factors, betas estimated from OLS regression, residual terms are random draws (diagonal covariance matrix plus correlation parameter (rho = 0.5).												
1	0.412	0.213	-5.437	-5.062	0.466	0.212	-5.295	-4.826	0.120	0.107	-4.934	-4.586
2	0.133	0.168	2.614	3.442	0.178	0.183	2.419	1.940	0.104	0.088	-2.097	-2.098
3	0.057	0.079	3.276	2.859	0.078	0.081	1.903	1.789	0.093	0.077	-0.901	-1.648
4	0.046	0.066	1.387	1.368	0.053	0.048	1.534	1.709	0.074	0.065	0.569	-0.418
5	0.029	0.035	1.920	2.295	0.050	0.036	-0.546	-0.769	0.061	0.047	0.586	0.368

Factor	1986 - 1995				1986 - 1990				1991 - 1995			
	Mean	Stdev	t-stat[2]	z-stat[3]	Mean	Stdev	t-stat[2]	z-stat[3]	Mean	Stdev	t-stat[2]	z-stat[3]
Panel C: Model #3 - Factors, betas, and residuals are random draws from i.i.d. multivariate normal distributions. Parameters are tied to sample observations.												
1	0.671	0.030	-20.877	-12.110	0.680	0.041	-16.032	-11.216	0.526	0.056	-59.865	-12.217
2	0.256	0.056	-3.457	-3.964	0.182	0.058	2.870	-0.166	0.226	0.068	-14.574	-10.499
3	0.222	0.042	-8.292	-7.912	0.151	0.059	-4.079	-5.960	0.195	0.057	-11.658	-9.333
4	0.181	0.050	-12.414	-9.824	0.117	0.055	-5.817	-7.512	0.158	0.059	-8.356	-7.534
5	0.123	0.053	-12.132	-9.669	0.076	0.042	-5.097	-6.005	0.095	0.050	-4.134	-4.749
Panel D: Model #4 - Factors, betas, and residuals are random draws from i.i.d. multivariate normal distributions. Parameters are tied to sample observations. Betas are scaled (4x, 2x, 1x, 0.5x, 0.25x).												
1	0.935	0.010	-34.371	-12.217	0.933	0.012	-27.026	-12.217	0.913	0.015	-151.348	-12.217
2	0.703	0.041	-29.478	-12.212	0.664	0.048	-18.994	-11.479	0.686	0.052	-67.098	-12.217
3	0.267	0.057	-11.167	-9.013	0.205	0.066	-8.388	-8.028	0.228	0.075	-13.448	-9.918
4	0.017	0.013	5.194	6.612	0.026	0.015	5.799	7.208	0.025	0.016	7.419	7.311
5	0.014	0.007	5.735	6.848	0.023	0.013	5.846	6.232	0.022	0.015	8.045	8.164

Factor	1986 - 1995 Mean	Stdev	t-stat[2]	z-stat[3]	1986 - 1990 Mean	Stdev	t-stat[2]	z-stat[3]	1991 - 1995 Mean	Stdev	t-stat[2]	z-stat[3]
Panel E: Model #5 - Factors and betas are random draws from i.i.d. multivariate normal distributions. Residuals are drawn from multivariate normal based on covariance matrix (so approximate factor structure). Parameters are tied to sample observations.												
1	0.429	0.269	-5.261	-4.774	N/A				N/A			
2	0.224	0.187	-1.136	-1.135								
3	0.160	0.145	-2.681	-4.035								
4	0.149	0.144	-5.279	-6.133								
5	0.122	0.114	-6.736	-7.270								

Note: Panel E does not have results for the two five year intervals because Model #5 requires that the number of assets be less than the number of observations in the time-series of returns. Therefore I only look at the ten year period so that sample size can be reasonably large.

[1] p-value is for the Shapiro-Wilk W statistic (I report the probability since the statistic is not commonly used).
[2] t-stat is for Satterthwaite's approximate test of equality with unequal variances.
[3] z-stat is for Wilcoxon rank sum test.

TABLE 6-9

Comparing the R^2 Values From Real Data and Simulated Data - (n = 150)

This table compares the R^2 values obtained from the sample data to the R^2 values obtained from simulated data. For each model I use a standard t-test and a Wilcoxon rank sum test to see if the R^2 values from the real world data are significantly different than the R^2 values obtained under the assumed models. I compare the first five factors. The Wilcoxon test is performed because most of the sample R^2 values do not appear to be normally distributed (as indicated by a Shapiro-Wilk W-test). The real world data in panel A is the same as in Table 6.2 the R^2 values in the other panels are the mean and standard deviation across 100 iterations of regressing the factors extracted from one simulated portfolio of returns on the factors extracted from another simulated portfolio of returns.

	1986 - 1995			1986 - 1990			1991 - 1995		
Factor	Mean	Stdev	p-value[1]	Mean	Stdev	p-value[1]	Mean	Stdev	p-value[1]
Panel A: Real World Data.									
1	0.579	0.315	<0.001	0.706	0.270	<0.001	0.241	0.210	<0.001
2	0.210	0.278	<0.001	0.172	0.266	<0.001	0.200	0.175	<0.001
3	0.064	0.124	<0.001	0.049	0.034	<0.001	0.112	0.121	<0.001
4	0.022	0.017	<0.001	0.048	0.032	<0.001	0.094	0.118	<0.001
5	0.022	0.014	<0.001	0.049	0.031	<0.001	0.066	0.070	<0.001

	1986 - 1995				1986 - 1990				1991 - 1995			
Factor	Mean	Stdev	t-stat[2]	z-stat[3]	Mean	Stdev	t-stat[2]	z-stat[3]	Mean	Stdev	t-stat[2]	z-stat[3]
Panel B: Model #1 - extracted factors, betas estimated from OLS regression, residual terms are random draws (diagonal covariance matrix plus correlation parameter rho (rho = 0.5).												
1	0.608	0.331	-0.624	-2.735	0.784	0.195	-2.351	-4.977	0.361	0.251	-3.671	-3.861
2	0.234	0.318	-0.562	-7.621	0.161	0.218	0.321	-3.046	0.259	0.241	-1.983	-1.126
3	0.060	0.133	0.231	-0.237	0.077	0.054	-4.435	-4.178	0.106	0.126	0.366	0.182
4	0.031	0.028	-2.574	-2.822	0.069	0.051	-3.488	-3.247	0.067	0.054	2.097	-0.483
5	0.027	0.017	-2.148	-2.053	0.060	0.043	-2.147	-1.803	0.046	0.033	2.561	1.630

	1986 - 1995				1986 - 1990				1991 - 1995			
Factor	Mean	Stdev	t-stat[2]	z-stat[3]	Mean	Stdev	t-stat[2]	z-stat[3]	Mean	Stdev	t-stat[2]	z-stat[3]
Panel C: Model #3 - Factors, betas, and residuals are random draws from i.i.d. multivariate normal distributions. Parameters are tied to sample observations.												
1	0.869	0.011	-9.161	-12.137	0.874	0.016	-6.235	-10.920	0.790	0.028	-25.901	-12.217
2	0.568	0.037	-12.775	-7.621	0.502	0.048	-12.218	-8.696	0.564	0.052	-19.996	-10.990
3	0.535	0.038	-36.341	-11.660	0.450	0.047	-68.878	-12.217	0.518	0.050	-30.909	-11.958
4	0.510	0.039	-113.790	-12.217	0.403	0.058	-54.031	-12.217	0.476	0.048	-29.999	-11.592
5	0.467	0.044	-95.953	-4.847	0.348	0.064	-42.223	-12.211	0.417	0.061	-38.044	-12.070
Panel D: Model #4 - Factors, betas, and residuals are random draws from i.i.d. multivariate normal distributions. Parameters are tied to sample observations. Betas are scaled (4x, 2x, 1x, 0.5x, 0.25x).												
1	0.978	0.002	-12.628	-12.220	0.978	0.003	-10.095	-12.219	0.971	0.004	-34.763	-12.218
2	0.887	0.012	-24.371	-12.217	0.870	0.019	-26.177	-12.145	0.878	0.017	-38.622	-12.217
3	0.603	0.035	-41.812	-11.726	0.546	0.052	-79.940	-12.217	0.566	0.048	-34.806	-12.151
4	0.100	0.045	-16.226	-10.719	0.039	0.030	1.916	2.342	0.061	0.041	2.594	0.061
5	0.015	0.016	3.260	4.847	0.027	0.018	6.034	6.104	0.027	0.021	5.326	6.550

Factor	1986 - 1995				1986 - 1990				1991 - 1995			
	Mean	Stdev	t-stat[2]	z-stat[3]	Mean	Stdev	t-stat[2]	z-stat[3]	Mean	Stdev	t-stat[2]	z-stat[3]
Panel E: Model #5 - Factors and betas are random draws from i.i.d. multivariate normal distributions. Residuals are drawn from multivariate normal based on covariance matrix (so approximate factor structure). Parameters are tied to sample observations.												
1	0.799	0.178	-6.066	-8.468	N/A				N/A			
2	0.247	0.250	-0.983	-2.996								
3	0.249	0.223	-7.216	-8.435								
4	0.216	0.211	-9.142	-10.426								
5	0.188	0.177	-9.322	-10.221								

Note: Panel E does not have results for the two five year intervals because Model #5 requires that the number of assets be less than the number of observations in the time-series of returns. Therefore, I only look at the ten year period so that sample size can be reasonably large.

[1] p-value is for the Shapiro-Wilk W statistic (I report the probability since the statistic is not commonly used).
[2] t-stat is for Satterthwaite's approximate test of equality with unequal variances.
[3] z-stat is for Wilcoxon rank sum test.

TABLE 6-10

Comparing the R^2 Values From Real Data and Simulated Data - (n = 500)

This table compares the R^2 values obtained from the sample data to the R^2 values obtained from simulated data. For each model I use a standard t-test and a Wilcoxon rank sum test to see if the R^2 values from the real world data are significantly different than the R^2 values obtained under the assumed models. I compare the first five factors. The Wilcoxon test is performed because most of the sample R^2 values do not appear to be normally distributed (as indicated by a Shapiro-Wilk W-test). The real world data in panel A is the same as in Table 6.2 the R^2 values in the other panels are the mean and standard deviation across 100 iterations of regressing the factors extracted from one simulated portfolio of returns on the factors extracted from another simulated portfolio of returns.

	1986 - 1995			1986 - 1990			1991 - 1995		
Factor	Mean	Stdev	p-value[1]	Mean	Stdev	p-value[1]	Mean	Stdev	p-value[1]
Panel A: Real World Data.									
1	0.725	0.393	<0.001	0.929	0.045	<0.001	0.496	0.356	<0.001
2	0.231	0.388	<0.001	0.114	0.104	<0.001	0.281	0.304	<0.001
3	0.023	0.022	<0.001	0.105	0.111	<0.001	0.109	0.193	<0.001
4	0.034	0.030	<0.001	0.078	0.066	<0.001	0.057	0.103	<0.001
5	0.041	0.040	<0.001	0.089	0.057	<0.001	0.040	0.032	<0.001

	1986 - 1995				1986 - 1990				1991 - 1995			
Factor	Mean	Stdev	t-stat[2]	z-stat[3]	Mean	Stdev	t-stat[2]	z-stat[3]	Mean	Stdev	t-stat[2]	z-stat[3]
Panel B: Model #1 - extracted factors, betas estimated from OLS regression, residual terms are random draws (diagonal covariance matrix plus correlation parameter (rho = 0.5).												
1	0.743	0.377	-0.322	-2.368	0.940	0.033	-2.005	-2.363	0.586	0.326	-1.866	-2.905
2	0.288	0.356	-1.084	-6.457	0.390	0.173	-13.664	-10.475	0.367	0.296	-2.014	-3.776
3	0.132	0.096	-11.050	-9.497	0.319	0.140	-11.975	-9.978	0.137	0.141	-1.177	-5.093
4	0.086	0.076	-6.473	-6.235	0.256	0.144	-11.232	-9.124	0.092	0.088	-2.535	-6.136
5	0.087	0.080	-5.167	-5.538	0.135	0.085	-4.535	-4.188	0.075	0.052	-5.892	-6.022

Factor	1986 - 1995				1986 - 1990				1991 - 1995			
	Mean	Stdev	t-stat[2]	z-stat[3]	Mean	Stdev	t-stat[2]	z-stat[3]	Mean	Stdev	t-stat[2]	z-stat[3]
Panel C: Model #3 - Factors, betas, and residuals are random draws from i.i.d. multivariate normal distributions. Parameters are tied to sample observations.												
1	0.959	0.004	-5.942	-12.172	0.962	0.005	-7.220	-10.755	0.933	0.009	-12.267	-12.217
2	0.833	0.014	-15.484	-6.597	0.798	0.021	-64.214	-12.215	0.833	0.020	-18.078	-11.230
3	0.817	0.016	-294.514	-12.218	0.776	0.022	-59.389	-12.217	0.812	0.023	-36.134	-11.814
4	0.803	0.015	-232.293	-12.217	0.753	0.029	-93.703	-12.217	0.792	0.027	-68.931	-12.200
5	0.786	0.018	-169.680	12.213	0.717	0.038	-91.782	-12.217	0.761	0.024	-178.231	-12.217
Panel D: Model #4 - Factors, betas, and residuals are random draws from i.i.d. multivariate normal distributions. Parameters are tied to sample observations. Betas are scaled (4x, 2x, 1x, 0.5x, 0.25x).												
1	0.993	0.001	-6.819	-12.906	0.994	0.001	-14.400	-12.487	0.991	0.001	-13.918	-12.249
2	0.964	0.003	-18.866	-12.218	0.959	0.006	-80.795	-12.217	0.961	0.005	-22.343	-12.218
3	0.848	0.015	-312.508	-12.218	0.815	0.026	-62.406	-12.217	0.838	0.021	-37.520	-11.983
4	0.459	0.048	-75.711	-12.217	0.276	0.085	-18.404	-10.982	0.348	0.070	-23.312	-11.694
5	0.009	0.006	7.712	12.217	0.027	0.024	10.011	9.217	0.020	0.013	5.491	5.464

Note: There is no Panel E (Model #5) for this sample size.

[1] p-value is for the Shapiro-Wilk W statistic (I report the probability since the statistic is not commonly used).
[2] t-stat is for Satterthwaite's approximate test of equality with unequal variances.
[3] z-stat is for Wilcoxon rank sum test.

TABLE 6-11

Comparing the R^2 Values From Real Data and Simulated Data - (n = 1500)*

This table compares the R^2 values obtained from the sample data to the R^2 values obtained from simulated data. For each model I use a standard *t*-test and a Wilcoxon rank sum test to see if the R^2 values from the real world data are significantly different than the R^2 values obtained under the assumed models. I compare the first five factors. The Wilcoxon test is performed because most of the sample R^2 values do not appear to be normally distributed (as indicated by a Shapiro-Wilk *W*-test). The real world data in panel A is the same as in Table 6.2 the R^2 values in the other panels are the mean and standard deviation across 100 iterations of regressing the factors extracted from one simulated portfolio of returns on the factors extracted from another simulated portfolio of returns.

	1986 - 1995			1986 - 1990			1991 - 1995		
Factor	Mean	Stdev	*p*-value[1]	Mean	Stdev	*p*-value[1]	Mean	Stdev	*p*-value[1]
Panel A: Real World Data.									
1	0.557	0.469	<0.000	0.983	0.002	0.2944	0.640	0.432	<0.000
2	0.438	0.465	<0.000	0.261	0.269	<0.000	0.322	0.433	<0.000
3	0.044	0.056	<0.000	0.338	0.233	<0.000	0.020	0.012	<0.000
4	0.091	0.092	<0.000	0.283	0.186	<0.000	0.023	0.016	<0.000
5	0.097	0.088	<0.000	0.254	0.149	<0.000	0.031	0.024	<0.000

	1986 - 1995				1986 - 1990				1991 - 1995			
Factor	Mean	Stdev	*t*-stat[2]	*z*-stat[3]	Mean	Stdev	*t*-stat[2]	*z*-stat[3]	Mean	Stdev	*t*-stat[2]	*z*-stat[3]
Panel B: Model #1 - extracted factors, betas estimated from OLS regression, residual terms are random draws (diagonal covariance matrix plus correlation parameter (rho = 0.5).												
1	0.594	0.457	-0.557	-2.269	0.984	0.002	-5.136	-5.200	0.632	0.412	0.123	-3.592
2	0.516	0.390	-1.282	-3.905	0.518	0.299	-6.381	-6.862	0.579	0.307	-4.839	-6.528
3	0.222	0.165	-10.182	-8.943	0.718	0.125	-14.390	-10.382	0.383	0.134	-26.945	-12.176
4	0.214	0.169	-6.404	-6.181	0.713	0.079	-21.325	-11.734	0.322	0.167	-17.875	-11.946
5	0.108	0.116	0.743	-0.414	0.420	0.288	-5.108	-3.422	0.163	0.119	-10.851	-9.937

Factor	1986 - 1995				1986 - 1990				1991 - 1995			
	Mean	Stdev	t-stat[2]	z-stat[3]	Mean	Stdev	t-stat[2]	z-stat[3]	Mean	Stdev	t-stat[2]	z-stat[3]
Panel C: Model #3 - Factors, betas, and residuals are random draws from i.i.d. multivariate normal distributions. Parameters are tied to sample observations.												
1	0.979	0.002	-8.990	-12.164	0.987	0.002	-15.910	-10.741	0.976	0.003	-7.787	-12.220
2	0.910	0.007	-10.154	-1.466	0.926	0.009	-24.675	-12.217	0.939	0.007	-14.231	-8.786
3	0.902	0.007	-151.955	-12.217	0.916	0.010	-24.820	-12.217	0.933	0.008	-629.240	-12.217
4	0.896	0.008	-87.154	-12.217	0.905	0.011	-33.425	-12.217	0.921	0.009	-482.730	-12.217
5	0.885	0.010	-88.386	-12.217	0.891	0.015	-42.436	-12.217	0.911	0.011	-330.388	-12.217
Panel D: Model #4 - Factors, betas, and residuals are random draws from i.i.d. multivariate normal distributions. Parameters are tied to sample observations. Betas are scaled (4x, 2x, 1x, 0.5x, 0.25x).												
1	0.997	0.000	-9.367	-12.980	0.998	0.000	-76.586	-12.959	0.997	0.000	-8.271	-12.930
2	0.982	0.002	-11.701	-12.226	0.986	0.002	-26.921	-12.237	0.987	0.002	-15.335	-12.227
3	0.921	0.008	-155.243	-12.217	0.934	0.009	-25.606	-12.217	0.941	0.008	-629.243	-12.217
4	0.663	0.029	-59.256	-12.217	0.650	0.045	-19.204	-11.306	0.705	0.040	-156.150	-12.217
5	0.029	0.024	-7.399	-7.068	0.021	0.015	15.515	11.913	0.026	0.021	1.469	1.845

* Note: Due to sample size limitations n = 1000 for the full ten year period. There is no Panel E (Model #5) for this sample size.

[1] p-value is for the Shapiro-Wilk W statistic (I report the probability since the statistic is not commonly used).
[2] t-stat is for Satterthwaite's approximate test of equality with unequal variances.
[3] z-stat is for Wilcoxon rank sum test.

TABLE 6-12
Dispersion of the Beta Coefficients in OLS Regressions

This table reports the mean and standard deviation of the beta estimates from the time-series OLS regressions. These estimates betas are used as the independent variables in the cross-sectional regressions to estimate the price of each risk factor. The sorting schemes are intended to provide an adequate level of dispersion in the betas. Standard deviations are reported in parentheses. The results are shown for the overall ten year period.

Sorting Scheme	Intercept (s.d.)	β_{MP} (s.d.)	β_{UI} (s.d.)	β_{DEI} (s.d.)	β_{URP} (s.d.)	β_{UTS} (s.d.)	β_{EW} (s.d.)
Panel A: All variables							
Size	0.0047 (0.006)	-0.2084 (0.7934)	-0.8469 (2.9421)	1.4271 (5.7747)	0.2723 (1.1257)	0.1900 (0.3935)	0.7411 (0.2913)
Industry	-0.0029 (0.0067)	-0.0240 (0.7803)	-1.1844 (2.6395)	1.8404 (5.8293)	0.0975 (0.8150)	0.0649 (0.3442)	0.9280 (0.2326)
Panel B: No market index							
Size	0.0095 (0.0056)	0.2887 (0.8477)	-3.6696 (3.2870)	-1.9885 (5.6590)	1.5595 (1.2470)	0.6866 (0.3606)	
Industry	0.0029 (0.0060)	0.5852 (0.8123)	-4.4894 (2.9017)	-2.2992 (6.0380)	1.6194 (0.9039)	0.6618 (0.3114)	

Sorting Scheme	Intercept (s.d.)	β_{MP} (s.d.)	β_{UI} (s.d.)	β_{DEI} (s.d.)	β_{URP} (s.d.)	β_{UTS} (s.d.)	β_{EW} (s.d.)
Panel C: No URP							
Size	0.0147 (0.0084)	-0.0648 (0.9680)	-4.2906 (2.9935)	-4.0571 (6.1471)		-0.0644 (0.2782)	
Industry	0.0083 (0.0076)	0.3645 (0.8694)	-5.4976 (2.7389)	-3.9849 (6.0233)		-0.1156 (0.2341)	
Panel D: No UTS							
Size	0.0136 (0.0053)	-0.1383 (0.9167)	-4.002 (3.3132)	-4.2035 (5.7214)	-0.0258 (0.8389)		
Industry	0.0067 (0.0061)	0.2175 (0.8319)	-4.9818 (2.9259)	-4.3720 (6.2264)	0.0331 (0.8340)		
Panel E: Only market index							
Size	0.0053 (0.0059)						0.7554 (0.2792)
Industry	-0.0028 (0.0065)						0.9455 (0.2108)

TABLE 6-13
Pricing of the Macroeconomic Factors

This table reports the average price for each of the proxies across the 120 (or 60 in five year periods) monthly estimates obtained in the month-by-month cross sectional regressions. A standard *t*-statistic is computed to test the mean value for significant departures from zero (*t*-statistic is reported in parentheses). The prices are multiplied by 100.

Sorting Scheme	λ_0 (*t*-stat)	λ_{MP} (*t*-stat)	λ_{UI} (*t*-stat)	λ_{DEI} (*t*-stat)	λ_{URP} (*t*-stat)	λ_{UTS} (*t*-stat)	λ_{EW} (*t*-stat)
Panel A1: 1986 - 1995 - All variables							
Size	1.1205	0.0702	0.0329	-0.0014	-0.0853	-0.2033	-0.2592
	(2.745)	(1.249)	(1.943)	(-0.215)	(-0.585)	(-0.438)	(-0.496)
Industry	0.7875	0.0654	-0.0286	0.0016	-0.1452	0.1773	-0.0407
	(1.752)	(0.701)	(-0.830)	(0.122)	(-0.743)	(0.409)	(-0.082)
Panel A2: 1986 - 1990 - All variables							
Size	0.3960	0.0792	0.0394	-0.0120	-0.3182	0.2827	-0.1537
	(0.587)	(0.942)	(1.845)	(-1.403)	(-1.223)	(0.348)	(-0.167)
Industry	1.1873	0.1926	0.0031	0.0075	-0.0388	0.0554	1.0059
	(1.695)	(1.230)	(0.059)	(0.361)	(-0.114)	(0.076)	(1.489)
Panel A3: 1991 - 1995 - All variables							
Size	1.8450	0.0612	0.0263	0.0092	0.1477	-0.6894	-0.3647
	(4.127)	(0.813)	(0.997)	(0.965)	(1.158)	(-1.524)	(-0.725)
Industry	0.3877	0.0618	0.0603	0.0108	0.2516	0.4100	0.9245
	(0.686)	(0.617)	(1.384)	(0.635)	(1.293)	(0.848)	(1.286)

Sorting Scheme	λ_0 (*t*-stat)	λ_{MP} (*t*-stat)	λ_{UI} (*t*-stat)	λ_{DEI} (*t*-stat)	λ_{URP} (*t*-stat)	λ_{UTS} (*t*-stat)	λ_{EW} (*t*-stat)
Panel B1:	1986 - 1995 - No EW						
Size	1.0611 (2.595)	0.0724 (1.049)	0.0340 (1.812)	-0.0055 (-0.478)	-0.0292 (-0.140)	-0.3027 (-0.583)	
Industry	0.7397 (1.749)	-0.0045 (-0.047)	-0.0086 (-0.221)	0.0021 (0.155)	-0.2583 (-1.343)	0.1131 (0.267)	
Panel B2:	1986 - 1990 - No EW						
Size	0.2785 (0.401)	0.0811 (0.747)	0.0378 (1.355)	-0.0167 (-0.814)	-0.3079 (-0.798)	0.2123 (0.230)	
Industry	0.4370 (0.618)	0.0263 (0.159)	0.0573 (1.043)	0.0156 (0.747)	-0.1566 (-0.482)	-0.1693 (-0.244)	
Panel B3:	1991 - 1995 - No EW						
Size	1.8438 (4.463)	0.0638 (0.740)	0.0302 (1.192)	0.0056 (0.515)	0.2499 (1.718)	-0.8178 (-1.723)	
Industry	1.0425 (2.233)	-0.0353 (-0.359)	-0.0745 (-1.377)	-0.0115 (-0.684)	-0.3600 (-1.731)	0.3954 (0.810)	

Sorting Scheme	λ_0 (*t*-stat)	λ_{MP} (*t*-stat)	λ_{UI} (*t*-stat)	λ_{DEI} (*t*-stat)	λ_{URP} (*t*-stat)	λ_{UTS} (*t*-stat)	λ_{EW} (*t*-stat)
Panel C1: 1986 - 1995 - No UTS or EW							
Size	0.9315 (2.307)	0.0748 (0.892)	0.0373 (1.727)	-0.0051 (-0.416)	0.0032 (0.015)		
Industry	0.5892 (1.367)	0.0016 (0.017)	0.0084 (0.226)	0.0074 (0.544)	-0.2222 (-1.147)		
Panel C2: 1986 - 1990 - No UTS or EW							
Size	0.1096 (0.156)	0.0902 (0.681)	0.0303 (0.905)	-0.0171 (-0.776)	-0.2888 (-0.716)		
Industry	0.2755 (0.377)	0.0135 (0.083)	0.0696 (1.212)	0.0238 (1.099)	-0.1560 (-0.475)		
Panel C3: 1991 - 1995 - No UTS or EW							
Size	1.7534 (4.623)	0.0594 (0.572)	0.0443 (1.610)	0.0068 (0.598)	0.2952 (1.921)		
Industry	0.9029 (1.958)	-0.0103 (-0.110)	-0.0529 (-1.153)	-0.0090 (-0.552)	-0.2884 (-1.384)		

Sorting Scheme	λ_0 (t-stat)	λ_{MP} (t-stat)	λ_{UI} (t-stat)	λ_{DEI} (t-stat)	λ_{URP} (t-stat)	λ_{UTS} (t-stat)	λ_{EW} (t-stat)
Panel D1: 1986 - 1995 - No URP or EW							
Size	0.9279 (2.647)	0.1229 (1.293)	0.0374 (1.626)	-0.0064 (-0.493)		0.1419 (0.310)	
Industry	0.8215 (2.060)	-0.0405 (-0.408)	-0.0079 (-0.210)	-0.0020 (-0.144)		0.3368 (1.179)	
Panel D2: 1986 - 1990 - No URP or EW							
Size	0.2333 (0.392)	0.1287 (0.828)	0.0288 (0.772)	-0.0094 (-0.411)		0.2518 (0.287)	
Industry	0.4229 (0.622)	-0.1085 (-0.698)	0.0504 (0.819)	0.0040 (0.168)		-0.1984 (-0.437)	
Panel D3: 1991 - 1995 - No URP or EW							
Size	1.6225 (4.573)	0.1171 (1.057)	0.0459 (1.698)	-0.0034 (-0.2710)		0.0320 (0.1205)	
Industry	1.220 (2.930)	0.0275 (0.220)	-0.0663 (-1.541)	-0.0081 (-0.514)		0.872 (2.591)	

TABLE 6-14
Comparing the Price of Factors Across Samples

This table makes pair-wise comparisons of the times-series of factor prices estimated from mutually exclusive portfolios. The results from a paired t-test and a Wilcoxon signed rank test are provided. I indicate factors that were initially significant in the either of the samples with superscript numerals. The prices are multiplied by 100.

Sorting Scheme	λ_0^a (t-stat)^b [z-stat]^c	λ_{MP}^a (t-stat)^b [z-stat]^c	λ_{UI}^a (t-stat)^b [z-stat]^c	λ_{DEI}^a (t-stat)^b [z-stat]^c	λ_{URP}^a (t-stat)^b [z-stat]^c	λ_{UTS}^a (t-stat)^b [z-stat]^c	λ_{EW}^a (t-stat)^b [z-stat]^c
Panel A1: 1986 - 1995 - All variables							
Size	0.9592[2]	0.1208[1]	0.0296	0.0003	0.1426	0.4286	0.9899
	(1.900)	(1.149)	(0.872)	(0.016)	(0.839)	(1.202)	(1.898)
	[1.655]	[0.798]	[1.061]	[1.016]	[0.707]	[1.142]	[1.590]
Industry	0.4100[12]	0.0290	0.0593	0.0077	0.3402	0.5070	0.3961
	(0.903)	(0.245)	(1.637)	(0.509)	(2.016)	(1.419)	(0.836)
	[0.524]	[0.005]	[1.527]	[0.050]	[2.111]	[1.268]	[0.372]
Panel A2: 1991 - 1995 - All variables							
Size	0.5905[2]	0.1113[1]	0.0842[2]	0.0025	0.1788	0.0950	0.6943
	(0.820)	(0.846)	(1.989)	(0.134)	(1.021)	(0.222)	(0.950)
	[0.721]	[0.817]	[1.803]	[0.817]	[1.237]	[0.331]	[0.854]
Industry	0.9457[1]	0.2089	0.0780[2]	0.0049	0.3597[2]	0.6102	1.1008
	(1.394)	(1.412)	(1.549)	(0.242)	(1.860)	(1.326)	(1.546)
	[0.942]	[1.325]	[1.229]	[0.140]	[1.428]	[1.163]	[0.994]

Sorting Scheme	λ_0 [a] (t-stat)[b] [z-stat][c]	λ_{MP} [a] (t-stat)[b] [z-stat][c]	λ_{UI} [a] (t-stat)[b] [z-stat][c]	λ_{DEI} [a] (t-stat)[b] [z-stat][c]	λ_{URP} [a] (t-stat)[b] [z-stat][c]	λ_{UTS} [a] (t-stat)[b] [z-stat][c]	λ_{EW} [a] (t-stat)[b] [z-stat][c]
Panel B1: 1986 - 1995 - No EW							
Size	0.5657[12]	0.0736	0.0320	0.0011	0.1816	0.3729	
	(1.666)	(0.722)	(0.985)	(0.083)	(1.1220)	(1.169)	
	[1.623]	[0.542]	[0.889]	[1.296]	[1.265]	[1.239]	
Industry	0.4129[12]	0.0053	0.0535	0.0122	0.3900[2]	0.3741	
	(1.406)	(0.045)	(1.626)	(0.778)	(2.317)	(1.117)	
	[0.799]	[0.162]	[1.605]	[0.136]	[2.315]	[0.707]	
Panel B2: 1991 - 1995 - No EW							
Size	1.1081[12]	0.0934[1]	0.0653	0.0030	0.0939	0.3412[2]	
	(1.849)	(0.730)	(1.594)	(0.149)	(0.565)	(0.836)	
	[1.804]	[0.449]	[1.296]	[1.163]	[0.766]	[0.979]	
Industry	1.0606[12]	0.2345	0.0907[2]	0.0006	0.4183[2]	0.7354	
	(2.114)	(1.541)	(2.137)	(0.032)	(2.055)	(1.605)	
	[1.833]	[1.281]	[1.892]	[0.589]	[1.539]	[1.296]	

[a] The value reported is the absolute value of the difference between the two sample's estimate of the risk premium.
[b] *t*-stat is from a paired *t*-test.
[c] *z*-stat is from a Wilcoxon signed rank test.
[1] The risk premium was significant (at the 10% level) in the first sample.
[2] The risk premium was significant (at the 10% level) in the second sample.
[12] The risk premium was significant (at the 10% level) in the first and second sample.

APPENDIX B

FIGURES

FIGURE 6-1

First 25 Eigenvalues from 1986-1995 Sample

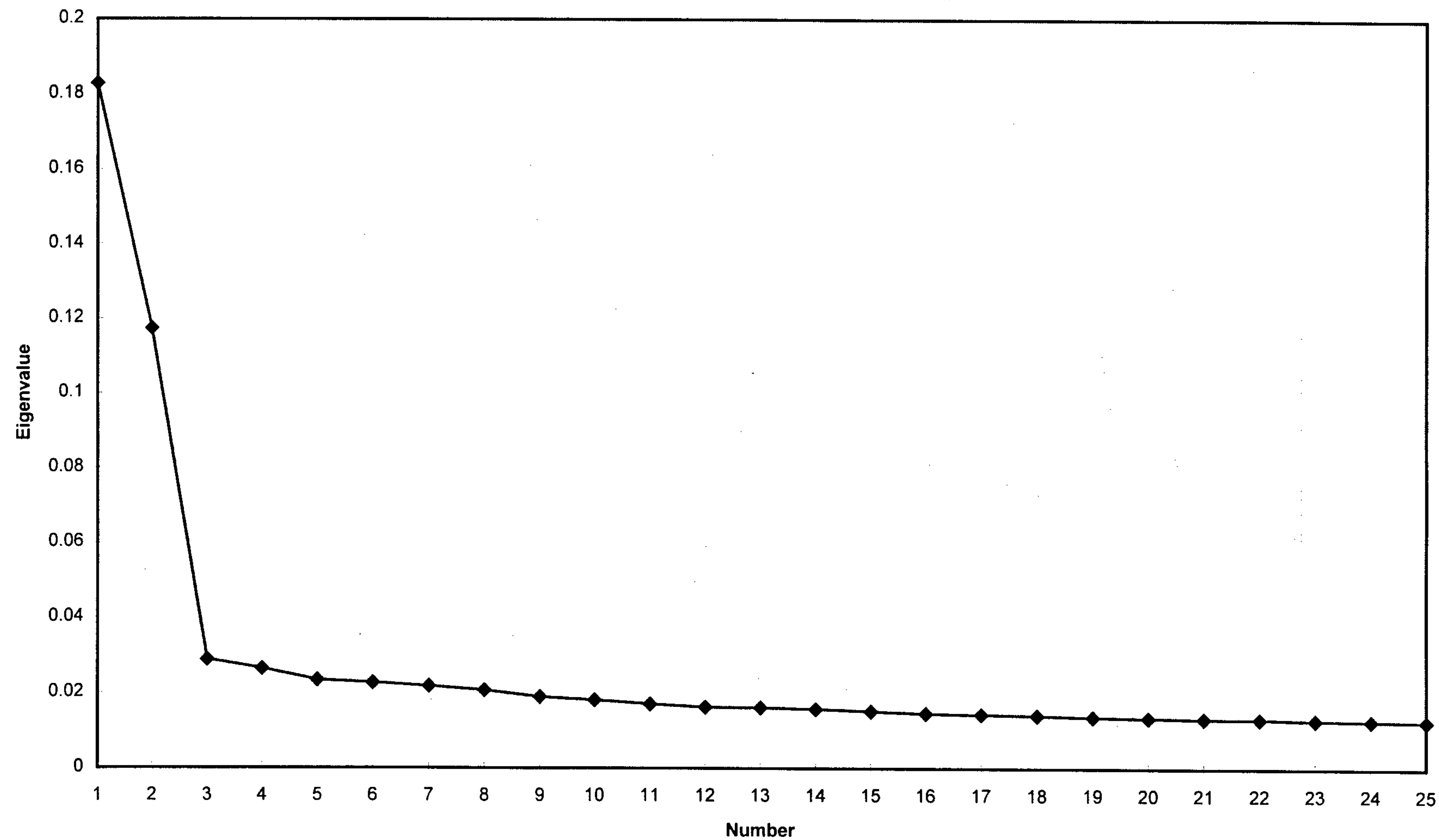


FIGURE 6-2
First 25 Eigenvalues from 1986-1990 Sample

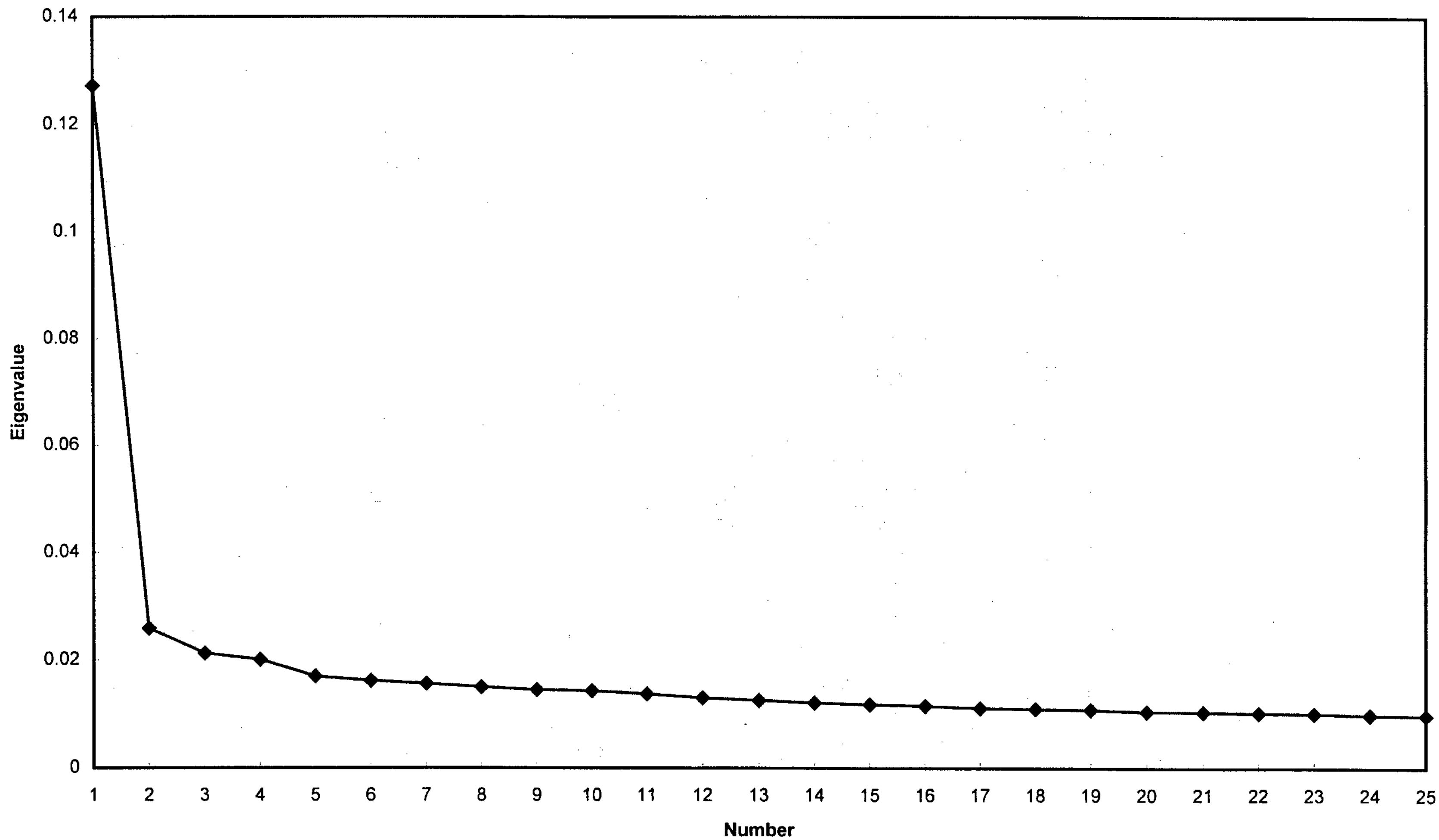
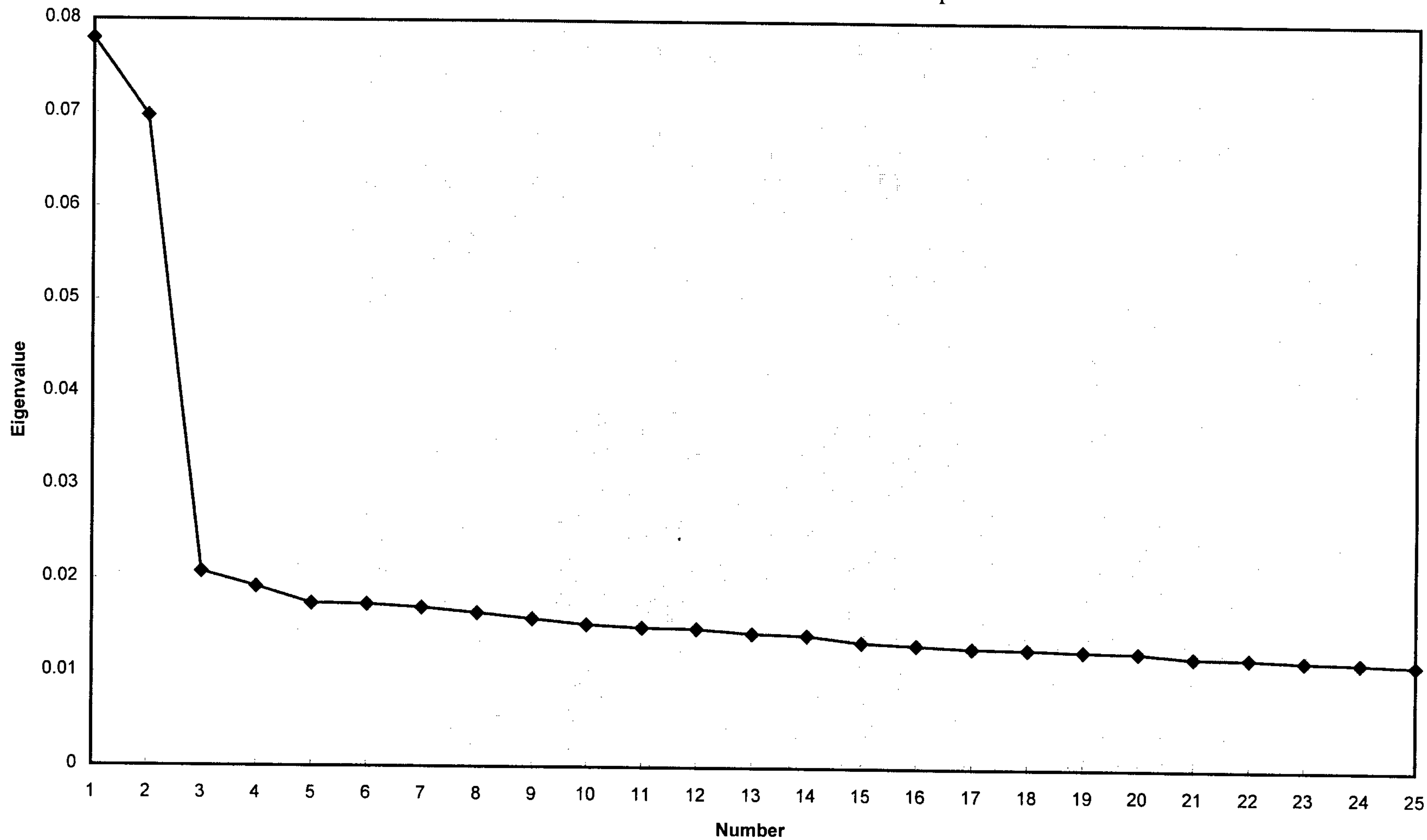


FIGURE 6-3
First 25 Eigenvalues from 1991-1995 Sample



APPENDIX C

INDUSTRY GROUPING

Industry Groupings by SIC Codes

This table provides the industry groupings as defined in Fama and French (1997). The first column reports the number of firms in each industry from my entire sample of 1680 securities.

#	Name	Industry	SIC Codes
8	Agric	Agriculture	0100-0799, 2048
36	Food	Food Products	2000-2046, 2050-2063, 2070-2079, 2090-2095, 2098-2099
6	Soda	Candy and Soda	2064-2068, 2086-2087, 2096-2097
7	Beer	Alcoholic Beverages	2080-2085
4	Smoke	Tobacco Products	2100-2199
18	Toys	Recreational Products	0900-0999, 3650-3652, 3732, 3930-3949
6	Fun	Entertainment	7800-7841, 7900-7999
23	Books	Printing and Publishing	2700-2749, 2770-2799
49	Hshld	Consumer Goods	2047, 2391-2392, 2510-2519, 2590-2599, 2840-2844, 3160-3199, 3229-3231, 3260, 3262-3263, 3269, 3630, 3630-3639, 3750-3751, 3800, 3860-3879, 3910-3919, 3960-3961, 3991, 3995
28	Clths	Apparel	2300-2390, 3020-3021, 3100-3111, 3130-3159, 3965
9	Hlth	Healthcare	8000-8099
28	MedEq	Medical Equipment	3693, 3840-3851
26	Drugs	Pharmaceutical Products	2830-2836
53	Chems	Chemicals	2800-2829, 2850-2899
19	Rubbr	Rubber and Plastic Products	3000, 3050-3099
15	Txtls	Textiles	2200-2295, 2297-2299, 2393-2395, 2397-2399
77	BldMt	Construction Materials	0800-0899, 2400-2439, 2450-2459, 2490-2499, 2950-2952, 3200-3219, 3240-3259, 3261, 3264, 3270-3299, 3420-3442, 3446-3452, 3490-3499, 3996
16	Cnstr	Construction	1500-1549, 1600-1699, 1700-1799
31	Steel	Steel Works, etc.	3300-3369, 3390-3399
12	FabPr	Fabricated Products	3400, 3443-3444, 3460-3479
84	Mach	Machinery	3510-3536, 3540-3569, 3580-3599
32	ElcEq	Electrical Equipment	3600-3621, 3623-3629, 3640-3646, 3648-3649, 3660, 3691-3692, 3699

#	Name	Industry	SIC Codes
11	Misc	Miscellaneous	3900, 3990, 3999, 9900-9999
33	Autos	Automobiles and Trucks	2296, 2396, 3010-3011, 3537, 3647, 3694, 3700-3716, 3790-3792, 3799
12	Aero	Aircraft	3720-3729
4	Ships	Shipbuilding, Railroad Equip	3730-3731, 3740-3743
4	Guns	Defense	3480-3489, 3760-3769, 3795
6	Gold	Precious Metals	1040-1049
16	Mines	Nonmetallic Mining	1000-1039, 1060-1099, 1400-1499
6	Coal	Coal	1200-1299
94	Energy	Petroleum and Natural Gas	1310-1389, 2900-2911, 2990-2999
153	Util	Utilities	4900-4999
16	Telcm	Telecommunications	4800-4899
9	PerSv	Personal Services	7020-7021, 7030-7039, 7200-7212, 7215-7299, 7395, 7500, 7520-7549, 7600-7699, 8100-8199, 8200-8299, 8300-8399, 8400-8499, 8600-8699, 8800-8899
87	BusSv	Business Services	2750-2759, 3993, 7300-7372, 7374-7394, 7397, 7399, 7510-7519, 8700-8748, 8900-8999
35	Comps	Computers	3570-3579, 3680-3689, 3695, 7373
93	Chips	Electronic Equipment	3622, 3661-3679, 3810, 3812
33	LabEq	Measuring and Control Equip	3811, 3820-3830
31	Paper	Business Supplies	2520-2549, 2600-2639, 2670-2699, 2760-2761, 3950-3955
14	Boxes	Shipping Containers	2440-2449, 2640-2659, 3210-3221, 3410-3412
34	Trans	Transportation	4000-4099, 4100-4199, 4200-4299, 4400-4499, 4500-4599, 4600-4699, 4700-4799
54	Whlsl	Wholesale	5000-5099, 5100-5199
64	Rtail	Retail	5200-5299, 5300-5399, 5400-5499, 5500-5599, 5600-5699, 5700-5736, 5900-5999
29	Meals	Restaurants, Hotel, Motel	5800-5813, 5890, 7000-7019, 7040-7049, 7213
43	Banks	Banking	6000-6099, 6100-6199
47	Insur	Insurance	6300-6399, 6400-6411
14	RlEst	Real Estate	6500-6553
151	Fin	Trading	6200-6299, 6700-6799